Drop deformation and breakup in laminar shear flow

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COST P21 Student Training School "Physics of droplets: Basic and Advanced topics"
Borovets, 13 July 2010

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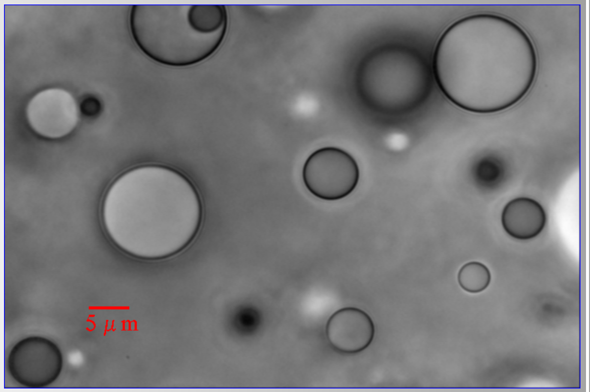
Background

- **Liquid-liquid biphasic systems**

  -Examples: emulsions, polymer blends, water-in-water biopolymer mixtures
  -In many cases system microstructure is characterized by **droplets** in a continuous phase



- **Applications**: detergents, personal care, food products, oil recovery

- **Flow-induced microstructure evolution**
  -Processing (e.g., mixing)
  -Final product properties and usage

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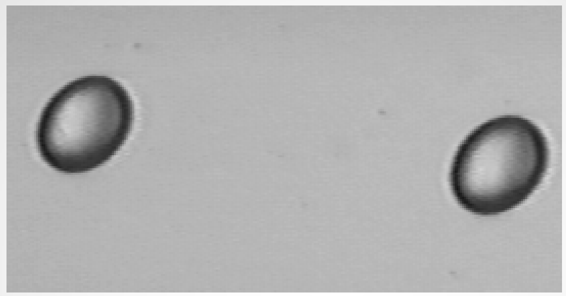
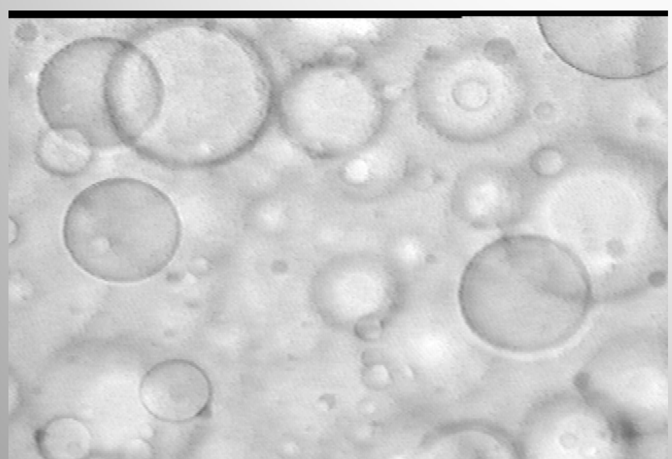
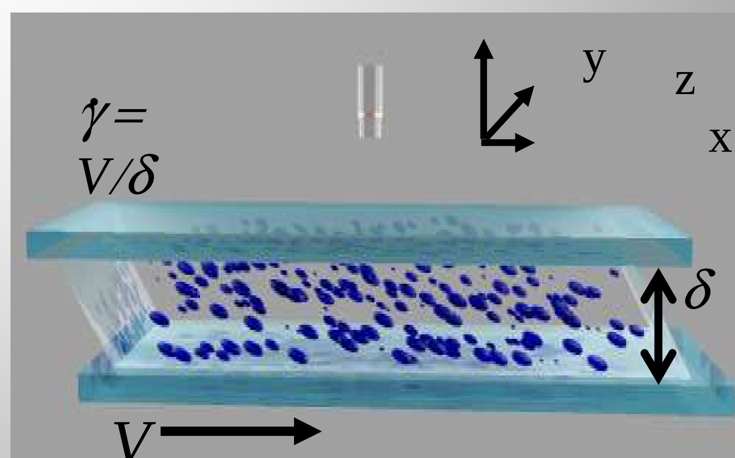
Background

- **Two main mechanisms governing microstructure dynamics under flow**
  -Droplet collision and coalescence



-Droplet deformation and breakup (this presentation)





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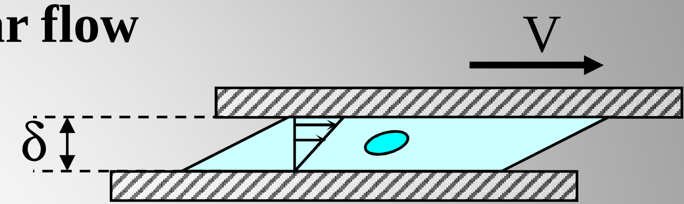
Topics

- **Drop deformation and breakup in simple shear flow**



$$\dot{\gamma} = \frac{V}{\delta}$$

–Isolated droplets

–No interfacial agents

–Immiscible, Newtonian fluids

–Laminar flow. However, results still apply in turbulent flow when eddies size is larger than droplet diameter (viscous turbulence)

Vankova N, Tcholakova S, Denkov ND, Ivanov IB, Vulchev VD, Danner Th. J Colloid Interface Sci, 312, 363 (2007)

- **Non-Newtonian effects**
- **Wall effects**
- **Effect of concentration**

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Reviews

1) Rallison J M, “The deformation of small viscous drops and bubbles in shear flows”, *Ann. Rev. Fluid Mech.*, **16**, 45-66 (1984).
2) Stone H A, “Dynamics of drop deformation and breakup in viscous fluids”, *Ann. Rev. Fluid Mech.*, **26** ,65-102 (1994) .
3) Tucker III C L and Moldenares P, “ Microstructural evolution in polymer blends”, *Ann. Rev. Fluid Mech.*, **34**, 177-210 (2002).
4) S. Guido and F. Greco, “Dynamics of a liquid drop in a flowing immiscible liquid”, in *Rheology Reviews 2004*, Ed. D. M. Binding and K. Walters, The British Society of Rheology, Aberystwyth, pp. 99-142 (2004)
5) Fischer P, Erni P, “Emulsion drops in external flow fields -- The role of liquid interfaces”, *Current Opinion in Colloid & Interface Science*, **12**,196-205 (2007).
6) Derkach, S R, “Rheology of emulsions”, *Adv Colloid Interface Sci*, **151**, 1-23 (2009)
7) Minale M, “Models for the deformation of a single ellipsoidal drop: a review”, *Rheol Acta,* online article
8) Frith W J, “Mixed Biopolymer Aqueous Solutions – Phase behaviour and rheology”, *Adv Colloid Interface Sci*, advanced online article
9) Guido S and Preziosi V, *Adv Colloid Interface Sci*, advanced online article

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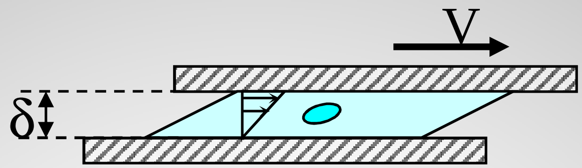
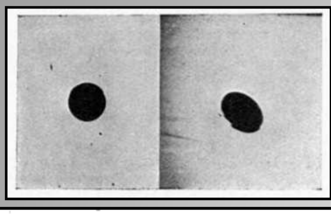
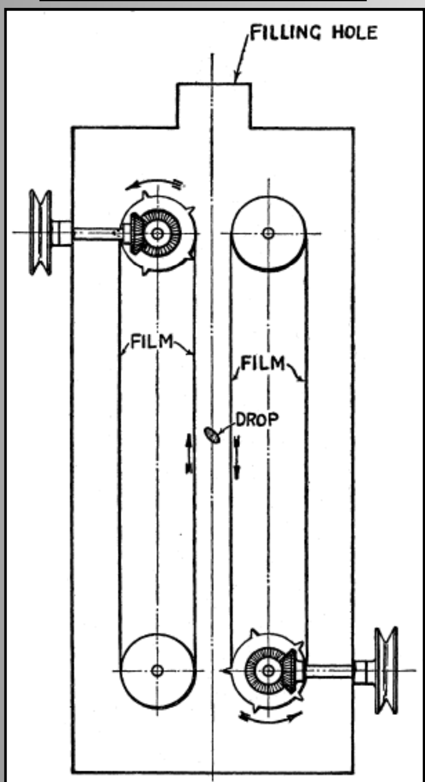
Experimental techniques



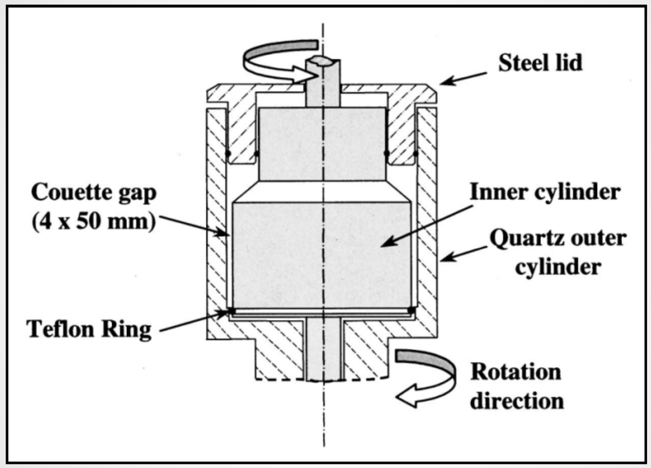
$$\dot{\gamma} = \frac{V}{\delta}$$





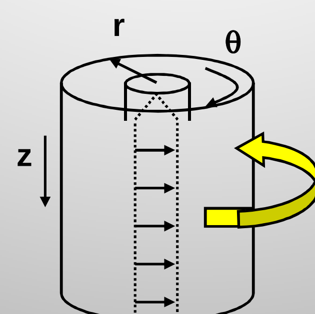
Parallel Band Apparatus

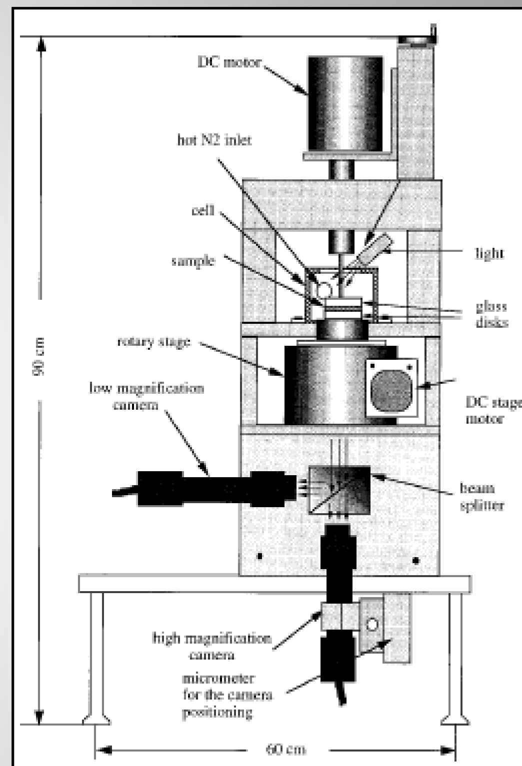
G. I. Taylor, *Proc. R. Soc. Lond. A*, **146**, 501-523 (1934)



Couette geometry

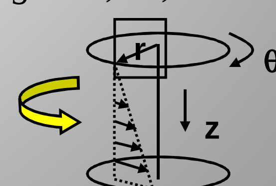
Mighri F and Huneault M A, *J. Rheol.*, **45,** 783-797 (2001)





Parallel plates (rotational)

Levitt L, Macosko C W and Pearson S D, *Polymer Eng. Sci.*, **36**, 1647-1655 (1996)



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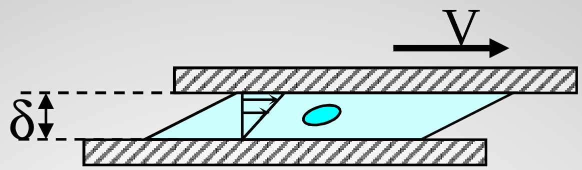
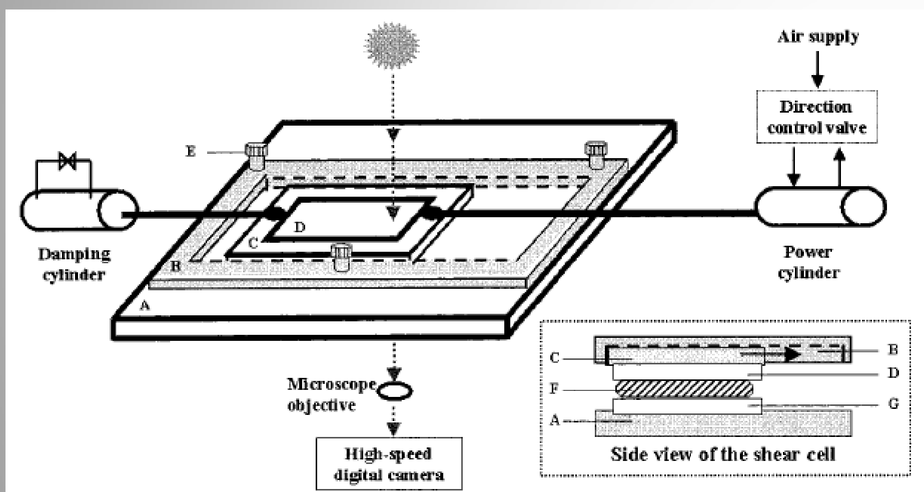
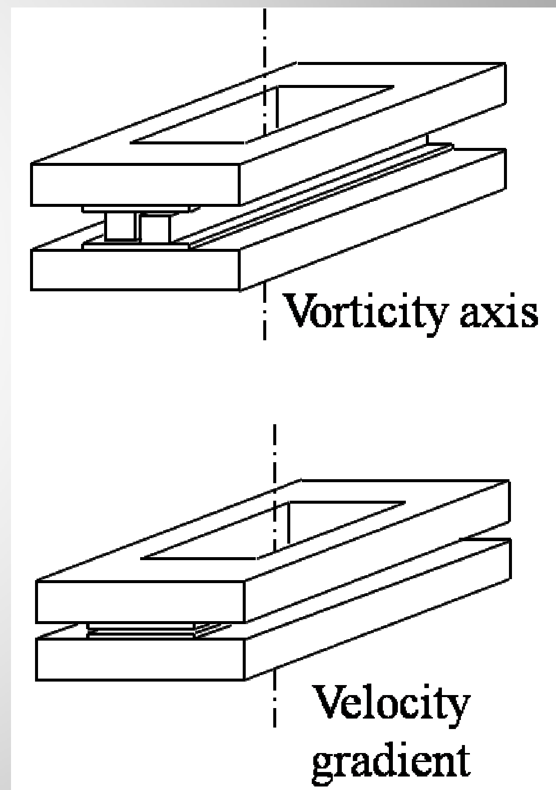
Experimental techniques



$$\dot{\gamma} = \frac{V}{\delta}$$



Parallel plates (translating, high-speed)
X. Zhao and J. L. Goveas, *Langmuir*, **17,** 3788-3791 (2001)



Parallel plates (translating)
S. Guido and M. Villone, *J. Rheology*, **42**, 395-415 (1998)

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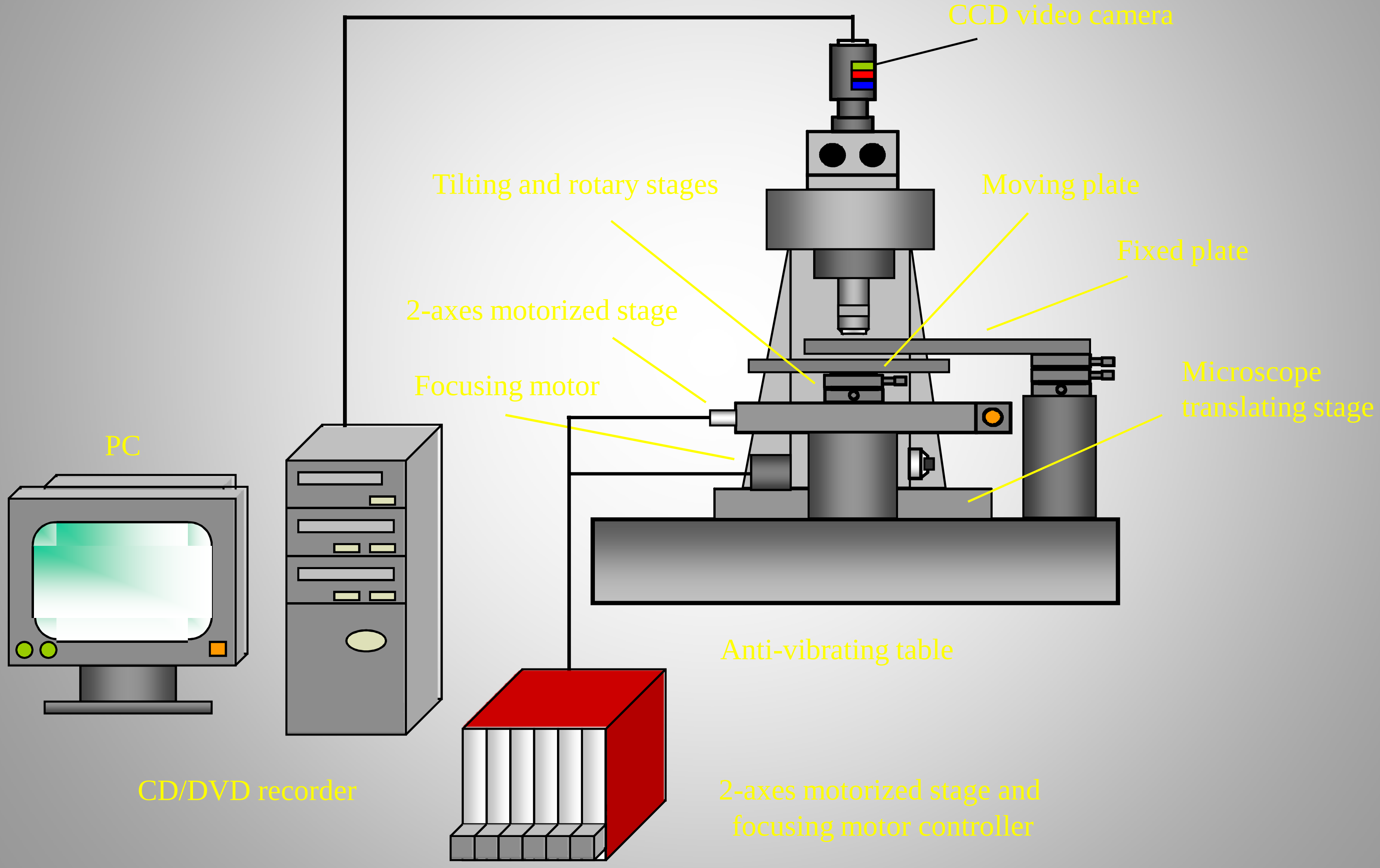
Shear flow workstation



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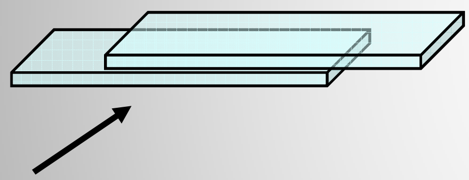
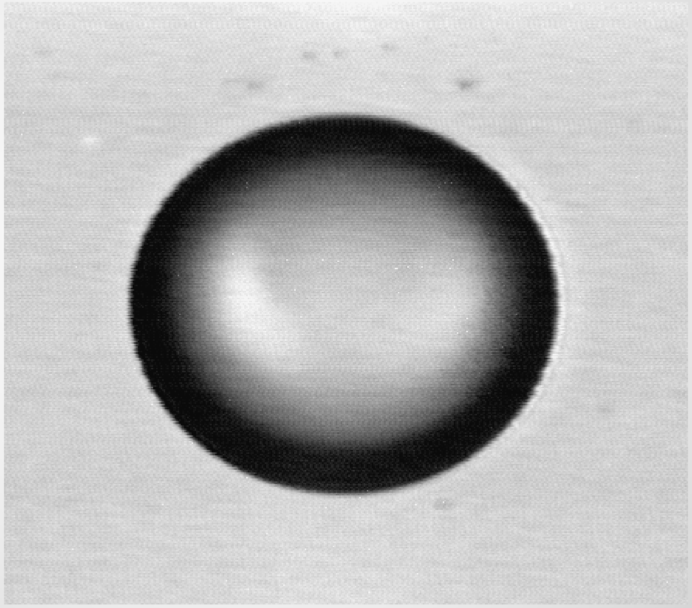
Drop shape at small deformations

View along vorticity
(video)





Example: drop of polydimethylsiloxane in polyisobutilene

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Small deformation theory

Newtonian case - Taylor, Chaffey-Brenner, Greco

Relevant physical quantities

η_c viscosity of continuous phase
η_d viscosity of drop phase
$\dot{\gamma}$ shear rate
r_0 drop radius at rest
σ interfacial tension

Nondimensional numbers

$$\mathrm{Ca} = \frac{\eta_c r_0 \dot{\gamma}}{\sigma}$$ Capillary number

$$\lambda = \frac{\eta_d}{\eta_c}$$ Viscosity ratio

$$\mathrm{Ca} = \frac{\tau_\sigma}{\tau_{\dot{\gamma}}}$$

Predictions

$$\begin{cases} \dfrac{r_{MAX}}{r_0} = 1 + f_1(\lambda)Ca + f_2(\lambda)Ca^2 \\ \dfrac{r_{MIN}}{r_0} = 1 - f_1(\lambda)Ca + f_2(\lambda)Ca^2 \\ \dfrac{r_z}{r_0} = 1 + f_3(\lambda)Ca^2 \end{cases}$$

$$\varphi = \frac{\pi}{4} + \frac{(19\lambda+16)(2\lambda+3)}{80(1+\lambda)}Ca$$

$$D \equiv \frac{r_{MAX} - r_{MIN}}{r_{MAX} + r_{MIN}} = \frac{19\lambda+16}{16\lambda+16}Ca$$

D ≡ Deformation parameter

G. I. Taylor, *Proc. R. Soc. Lond. A*, **146**, 501-523 (1934)
Chaffey CE, Brenner H, *J Colloid Interface Sci*, **24**, 258–269 (1967)
Greco F, *Phys. Fluids*, **14**, 946-954 (2002)

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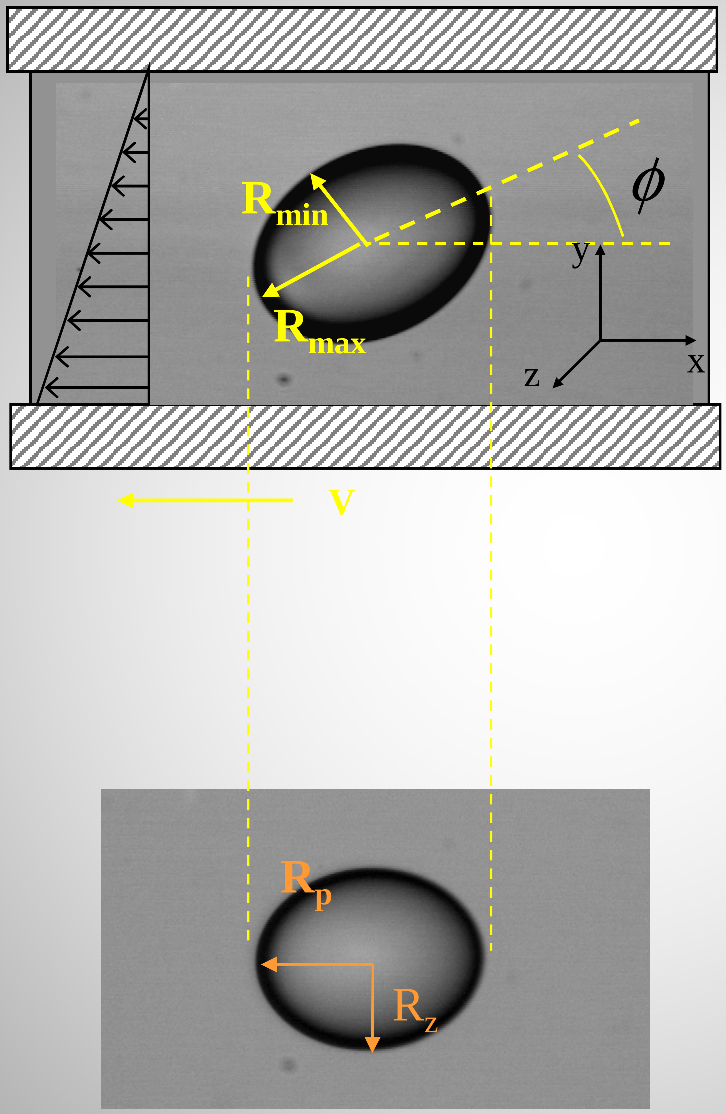
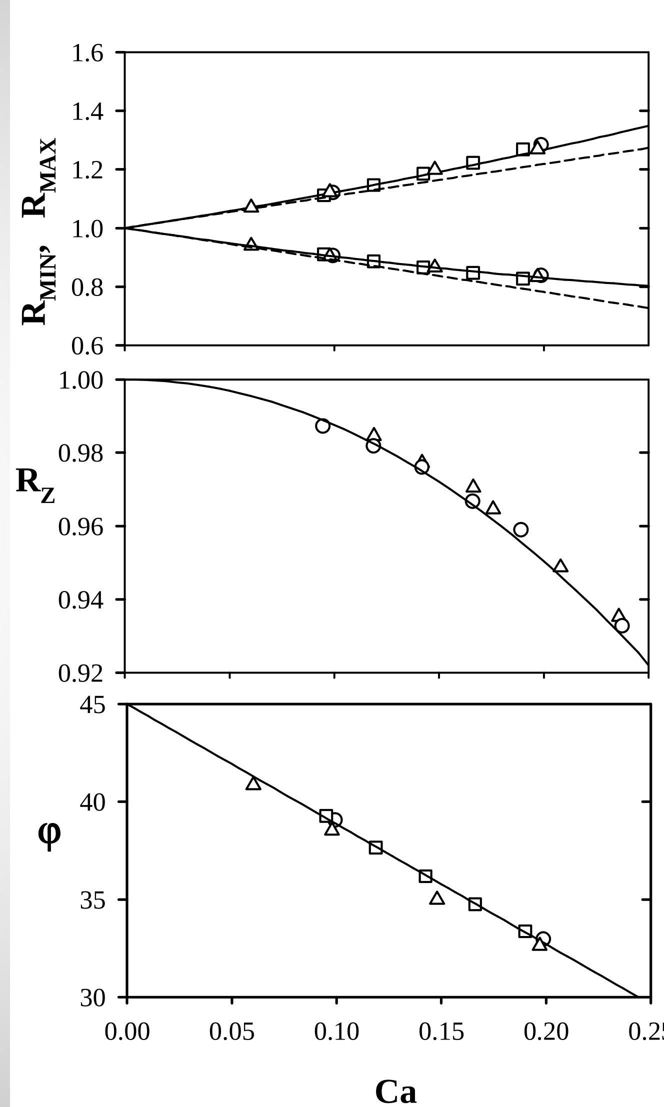
Stationary droplet shape





Drop shape is ellipsoidal up to moderate deformations

S. Guido and M. Villone, *J. Rheology*, **42**, 395-415 (1998)

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Ellipsoidal models

Maffettone-Minale model

$$\frac{d\mathbf{S}}{dt} - \mathrm{Ca}\left(\boldsymbol{\Omega}\cdot\mathbf{S} - \mathbf{S}\cdot\boldsymbol{\Omega}\right) = -f_1^{MM}\left[\mathbf{S} - g\left(\mathbf{S}\right)\mathbf{I}\right] + f_2^{MM}\mathrm{Ca}\left(\mathbf{D}\cdot\mathbf{S} + \mathbf{S}\cdot\mathbf{D}\right)$$

Ellipsoidal droplet described by a second order, positive-definite, symmetric tensor **S**

$\boldsymbol{\Omega} = 1/2\ \nabla\mathbf{v} - \nabla\mathbf{v}^{\mathbf{T}}$ and $\mathbf{D} = 1/2\ \nabla\mathbf{v} + \nabla\mathbf{v}^{\mathbf{T}}$, where $\nabla\mathbf{v}$ is the velocity gradient tensor

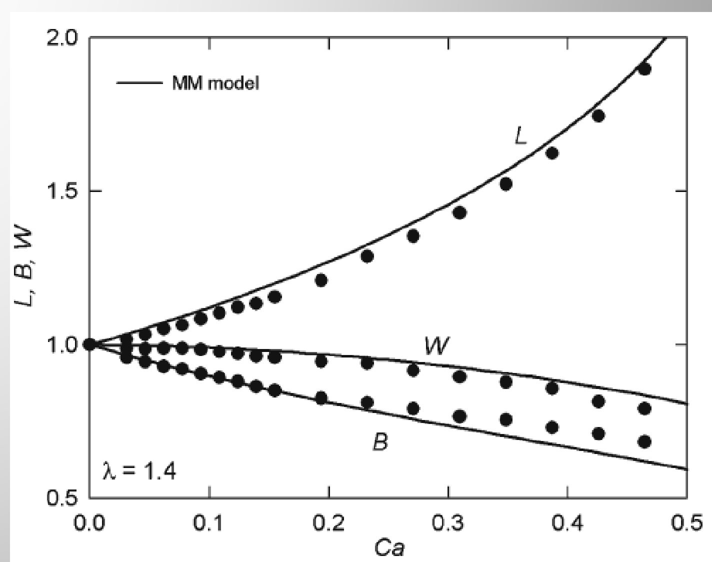
$g(\mathbf{S}) = 3III_S/II_S$, where III_S and II_S are the third and the second scalar invariant of **S** (to preserve droplet volume)

$$f_1^{\mathrm{MM}} = \frac{40\,(1+\lambda)}{(3+2\lambda)\,(16+19\lambda)};$$

$$f_2^{\mathrm{MM}} = \frac{5}{3+2\lambda} + \frac{3\mathrm{Ca}^2}{2+6\mathrm{Ca}^{2+\delta}}\,\frac{1}{1+\varepsilon\lambda^2},$$

f_i functions chosen to recover Taylor's small deformation limits

Maffettone PL, Minale M, *J Non-Newton Fluid Mech*, **78**, 227–241 (1998)



Other ellipsoidal models
Wetzel ED, Tucker CL III, *J Fluid Mech*, **426**,199–228 (2001)
Yu W, Bousmina M, Grmela M, Palierne J, Zhou C, *J Rheol*, **46**,1381–1399 (2002)
Edwards BJ, Dressler M, *Rheol Acta*, **42**,326–337 (2003)

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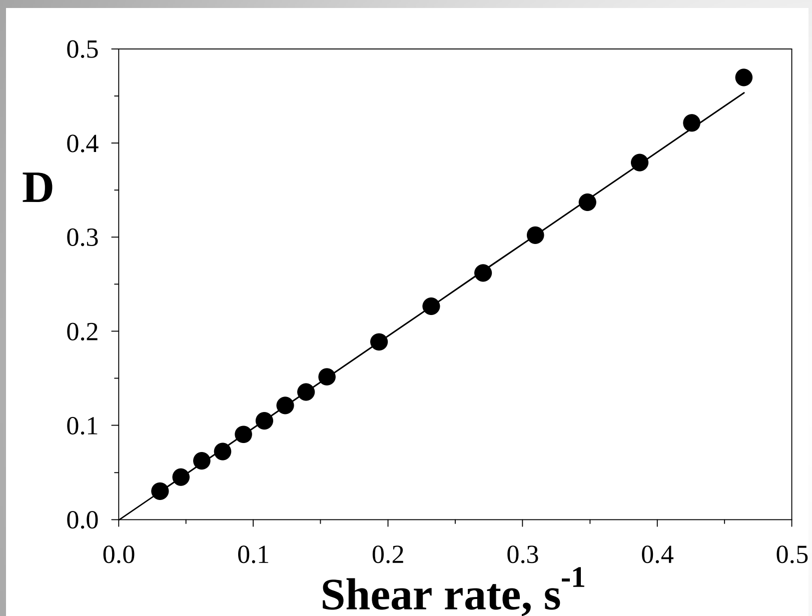
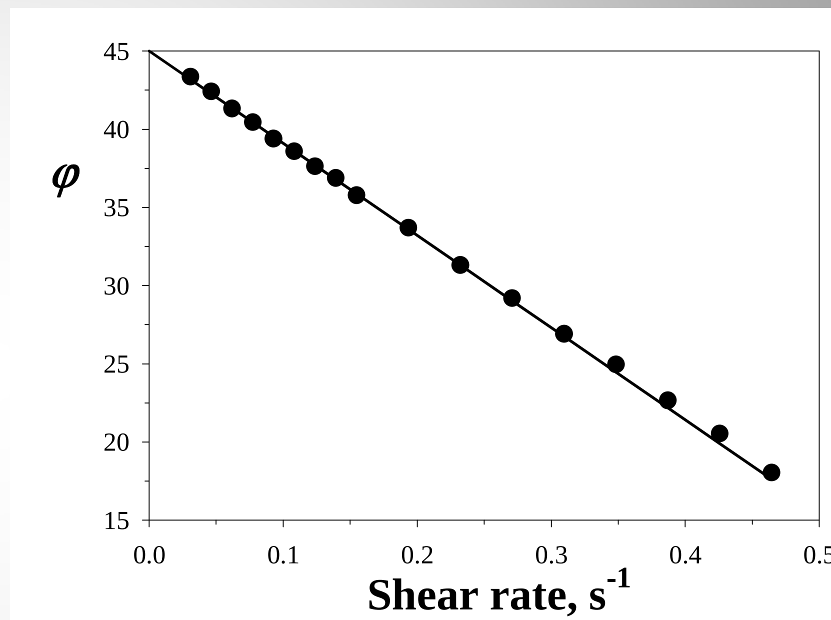
Interfacial tension measurement

FROM STEADY STATE SHAPE





$$D \equiv \frac{r_{MAX} - r_{MIN}}{r_{MAX} + r_{MIN}} = \frac{19\lambda + 16}{16\lambda + 16} Ca$$

$$\varphi = \frac{\pi}{4} + \frac{(19\lambda + 16)(2\lambda + 3)}{80(1+\lambda)} Ca$$

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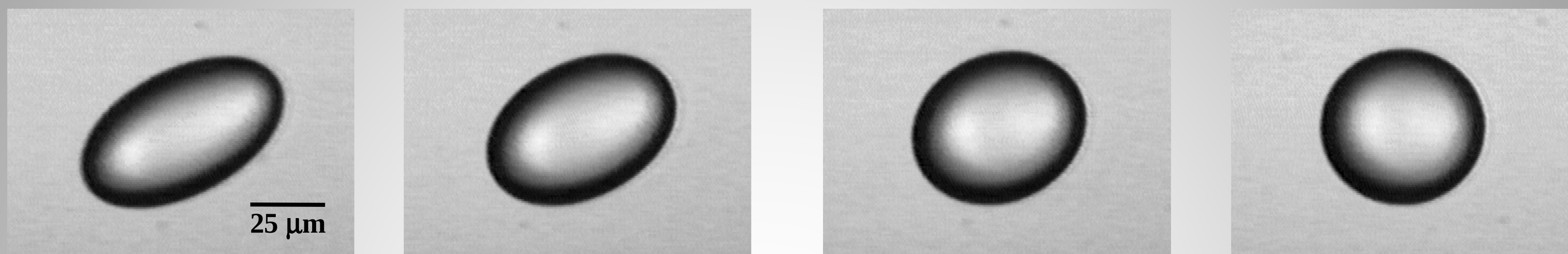
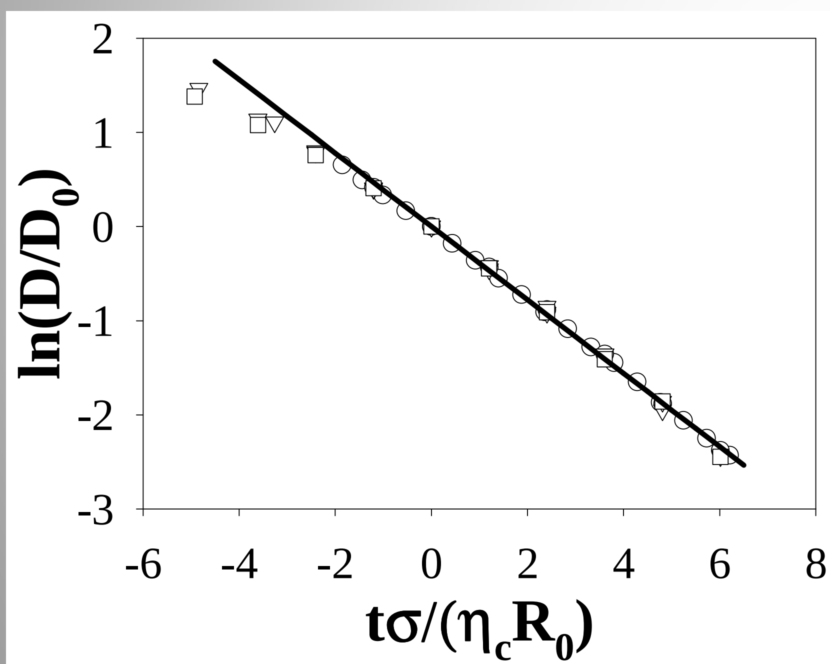
Interfacial tension measurement

FROM DROP RETRACTION





$$D = D_0 \exp\left(-\frac{40(\lambda+1)}{(2\lambda+3)(19\lambda+16)}\tau\right)$$

$$\tau = t\sigma/(\eta_c R_0)$$

Overall data

Method	σ (mN/m)
Steady state D, Taylor	2.54 ± 0.07
Steady state angle, Chaffey-Brenner	2.62 ± 0.07
Retraction, Rallison	2.58 ± 0.06

Luciani A, Champagne M F and Utracki L A,, *J. Polym. Sci. Phys. Ed.*, **35**, 1393-1403 (1997) .
Guido S and Villone M,, J. *Colloid Interface Sci.*, **209**, 247-250 (1999).

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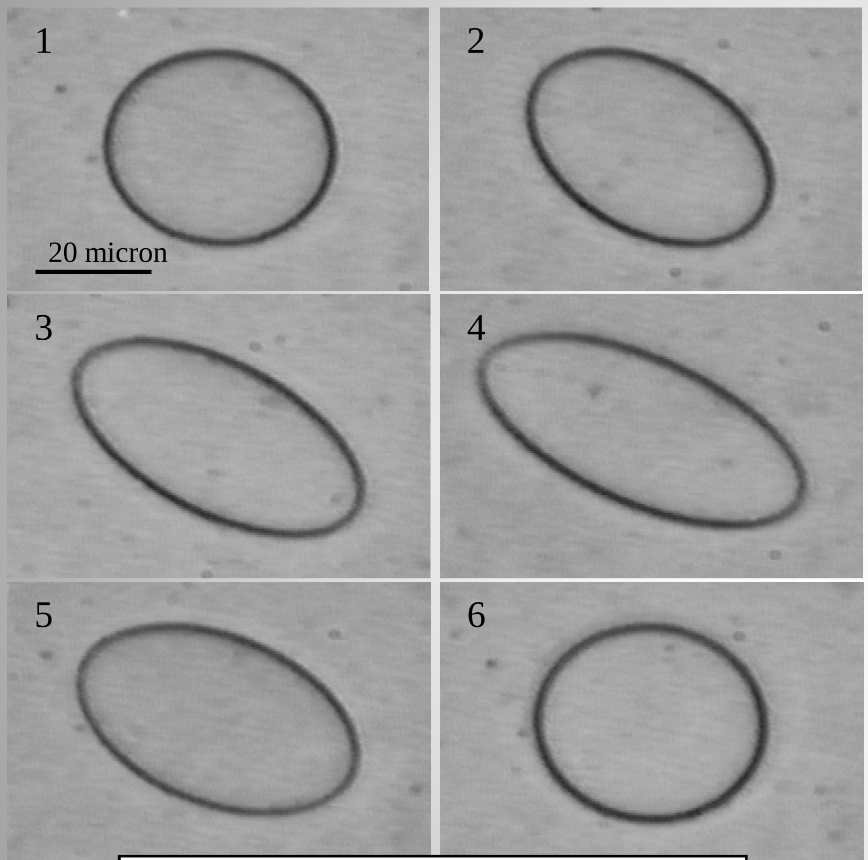
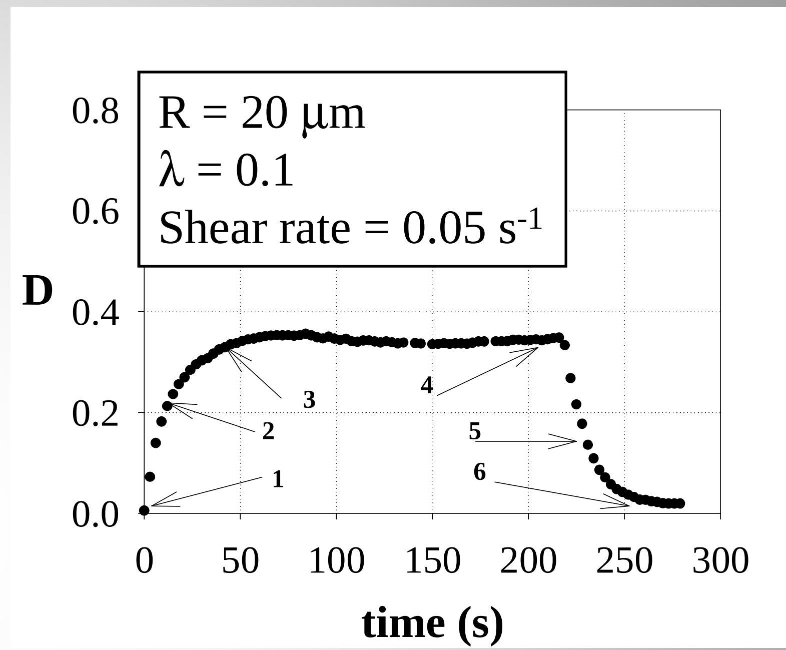
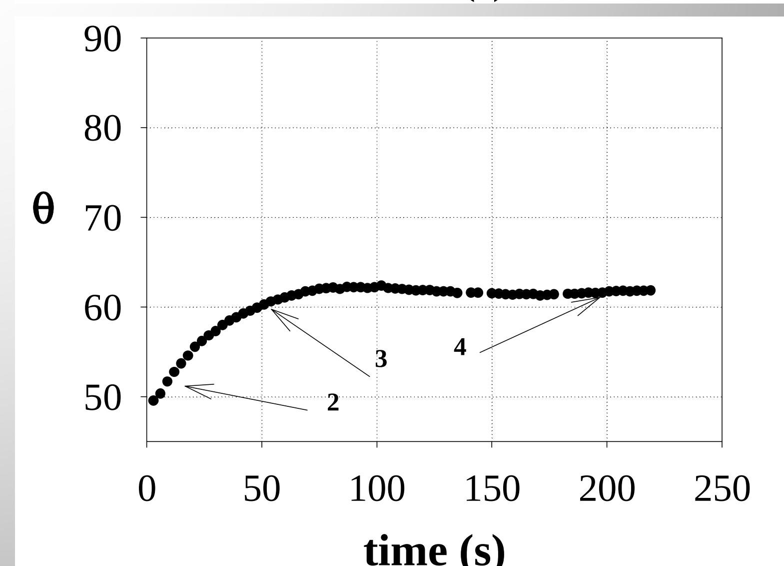
Water-in-water biopolymer mixtures



Drop: Na-caseinate rich phase
Matrix: Na-alginate rich phase





Guido, S Simeone M and Alfani, A, *Carbohydrate Polymers,* **48,** 143-152 (2002)

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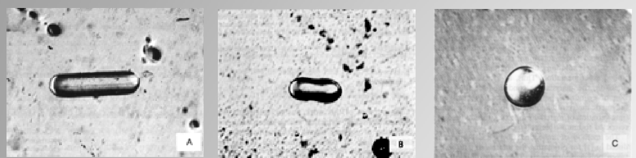
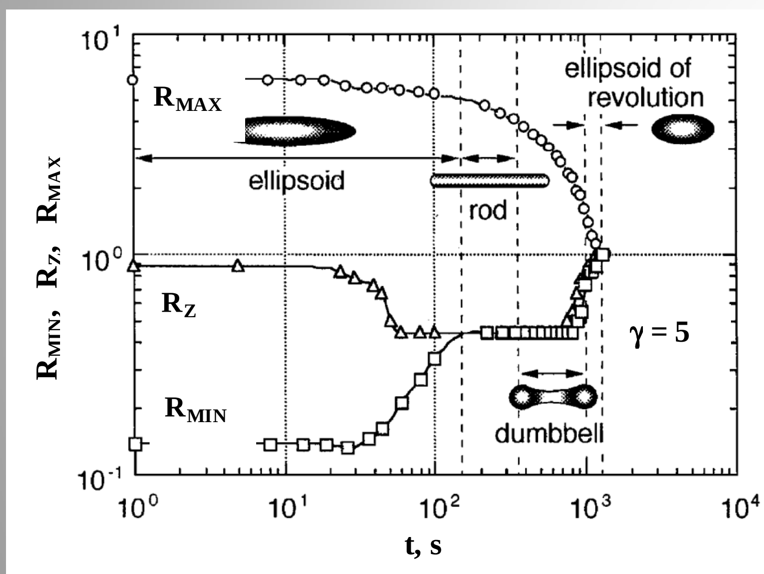
Droplet retraction after a step strain

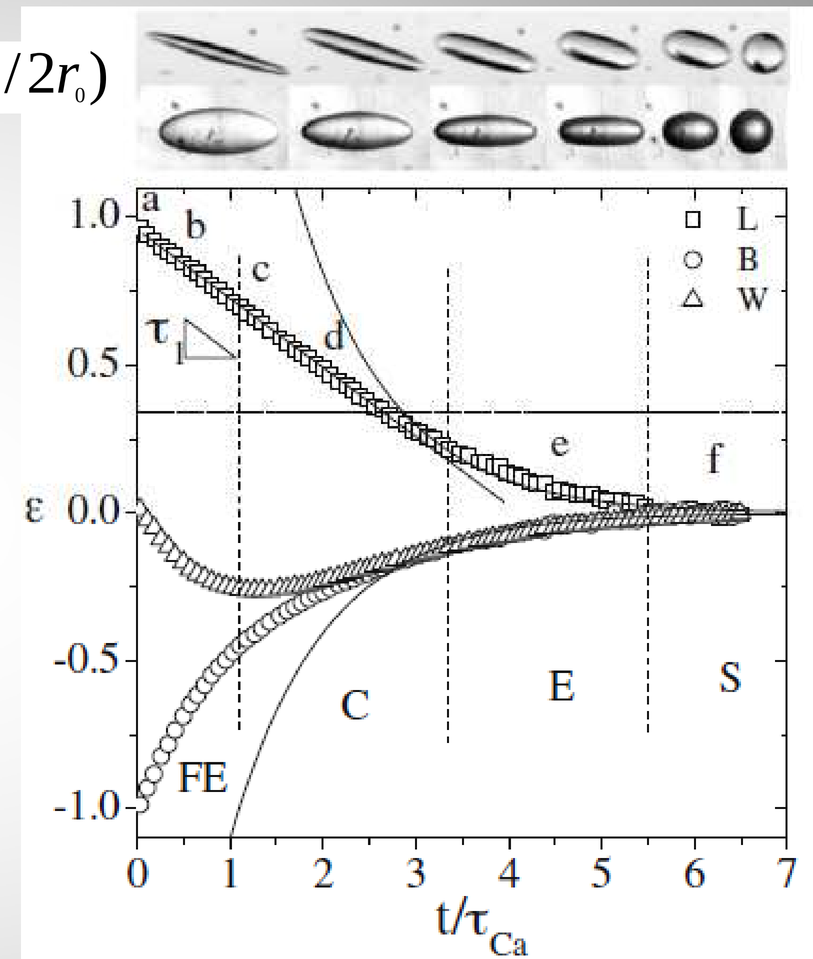


Cohen A and Carriere C J, *Rheol. Acta*, **28**. 223-232 (1989)



Yamane H, Takahashi M, Hayashi R, Okamoto K, Kashihara H and Masuda T, *J. Rheol.*, **42**, 567-580 (1998)

$$\varepsilon = \ln(L/2r_0)$$



FE flat ellipsoid, C spherocylinder,
E prolate ellipsoid, S sphere

Assighaou S and Benyahia L, *Rheol Acta*, **49**, 677-686 (2010)

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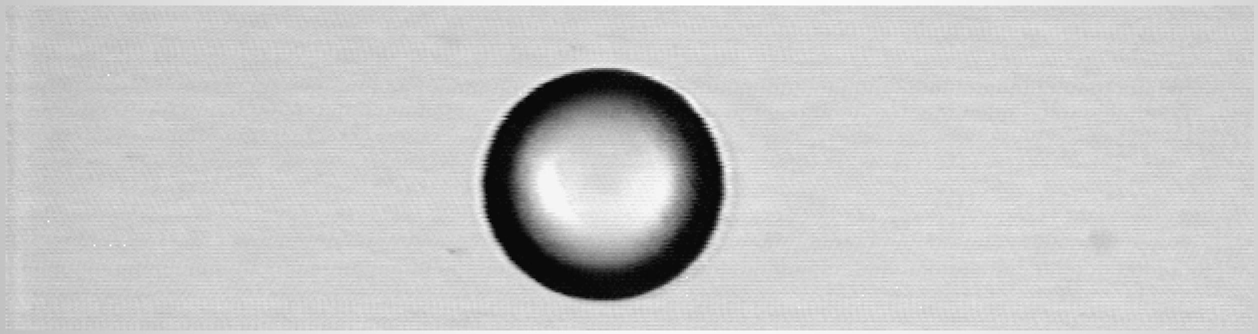
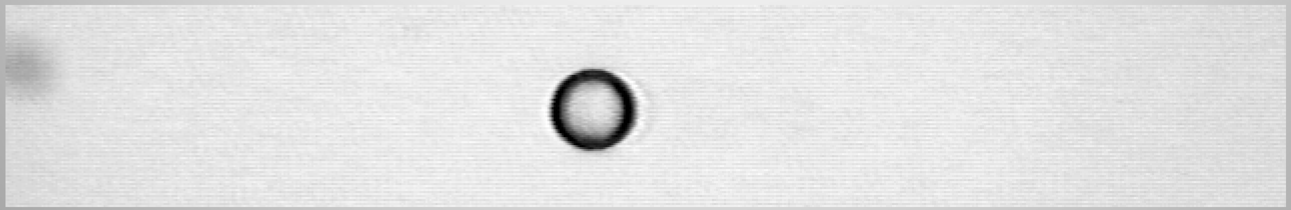
Drop breakup in shear flow

- Upon increasing *Ca*, a critical condition (Ca_{cr}) is reached where drop shape becomes unstable (video)



$\lambda = 1$

$Ca_1 > Ca_{cr}$



$\lambda = 1$

$Ca_2 > Ca_1$

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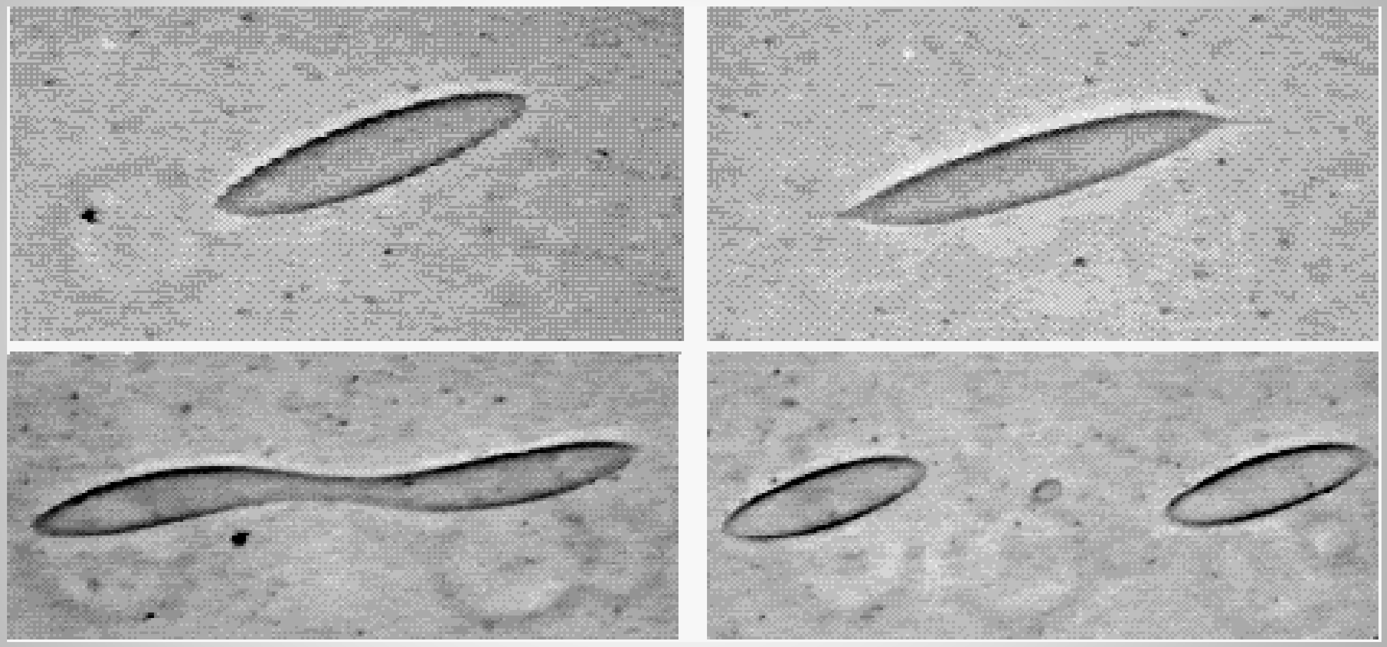
Drop breakup in shear flow

$\lambda \ll 1$



Tip streaming

Drop: Na-caseinate rich phase
Matrix: Na-alginate rich phase

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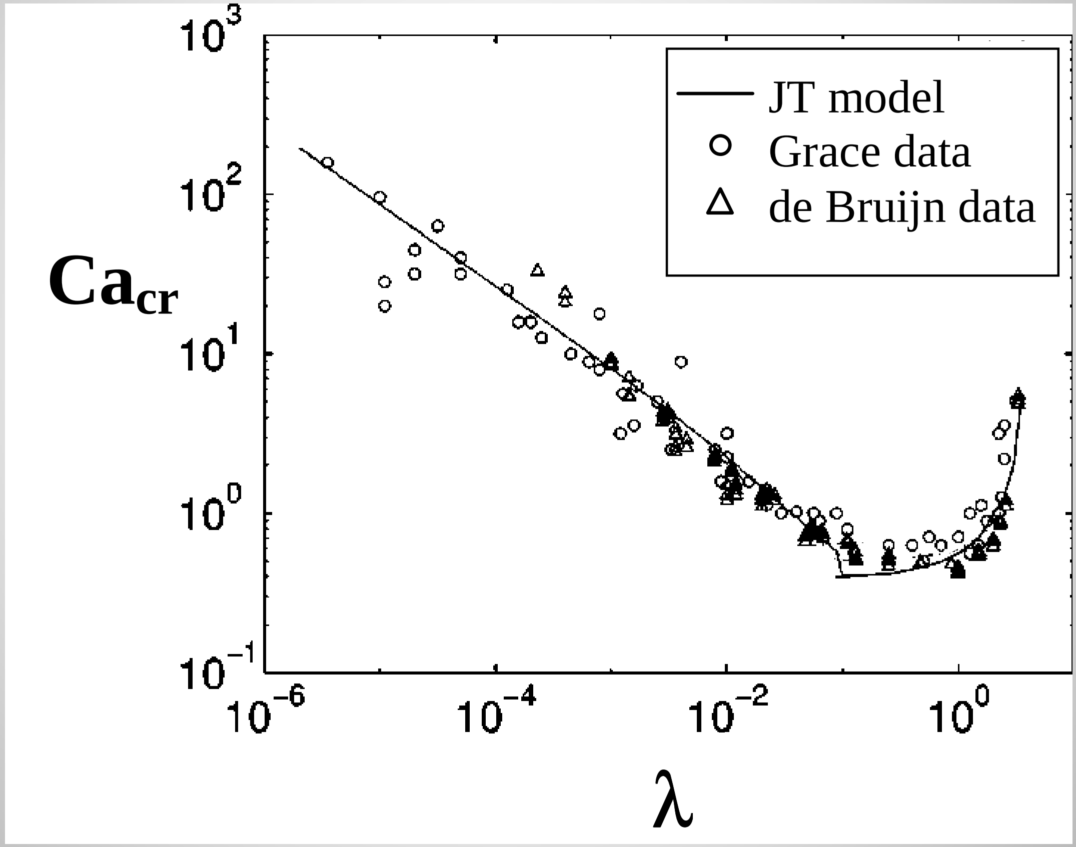
Drop breakup in shear flow



Grace H P, *Chem. Eng. Commun.*, **14**, 225-277 (1982)
de Bruijn R A, PhD thesis, Technische Universiteit Eindhoven (1989)
Jackson N E and Tucker III C L, *J. Rheol.*, **47**, 659-682 (2003)

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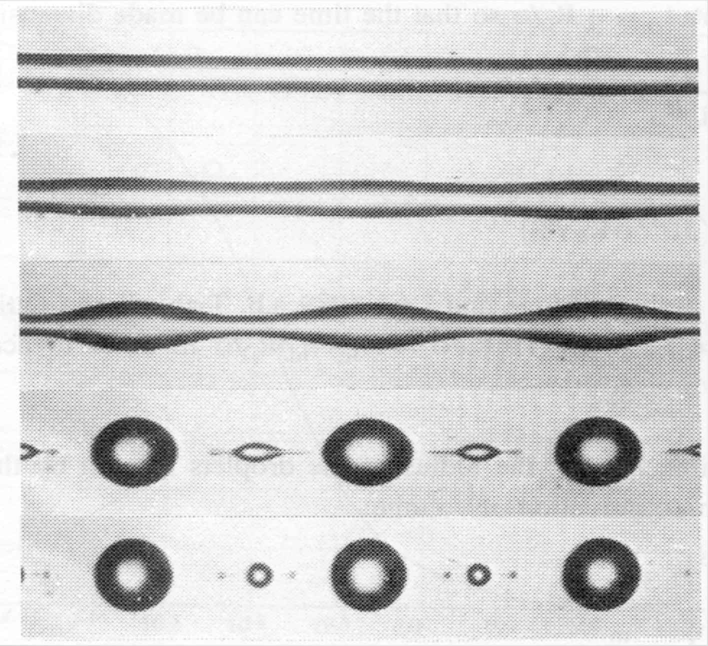
Breakup of a liquid thread



Tomotika S, **Proc. R. Soc. London Ser. A**, **150**, 322-337 (1935)
Elemans P H M, Janssen J M H and Meijer H E H, *J. Rheol.*, **34**, 1311-1325 (1990)

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Near critical behavior

Viscous-capillary force balance (inertia is negligible)

$$\eta\,\partial^2 v/\partial r^2 \approx \eta_d\,\partial^2 v/\partial z^2 \approx \sigma\,\partial r^{-1}/\partial r$$

Close to breakup, the external viscous shear stresses associated with thread axial motion become comparable to the internal viscous stresses associated with thread extension

$$\eta\frac{\partial v}{\partial r}$$

$$\eta_d\frac{\partial v}{\partial z}$$

$$\begin{cases} \dfrac{\zeta}{\ell} \approx \sqrt{\dfrac{\eta_d}{\eta}} \\ w \approx \dfrac{\sigma}{\eta} \end{cases}$$

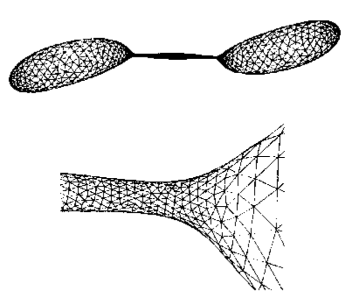
ζ characteristic axial length in the pinch-off region

l neck radius

w characteristic velocity

$$\zeta \equiv w(t_{cr} - t) \approx (\sigma/\eta)(t_{cr} - t) \;\Rightarrow\; \ell \approx \frac{\sigma}{\sqrt{\eta\eta_d}}(t_{cr} - t)$$

Lister J R and Stone H A, *Phys Fluids*, **10**, 2758-27 (1998)



Blawzdziewicz J, Cristini V and Loewenberg M, *Phys Fluids*, **14**, 2709-2718 (2002)

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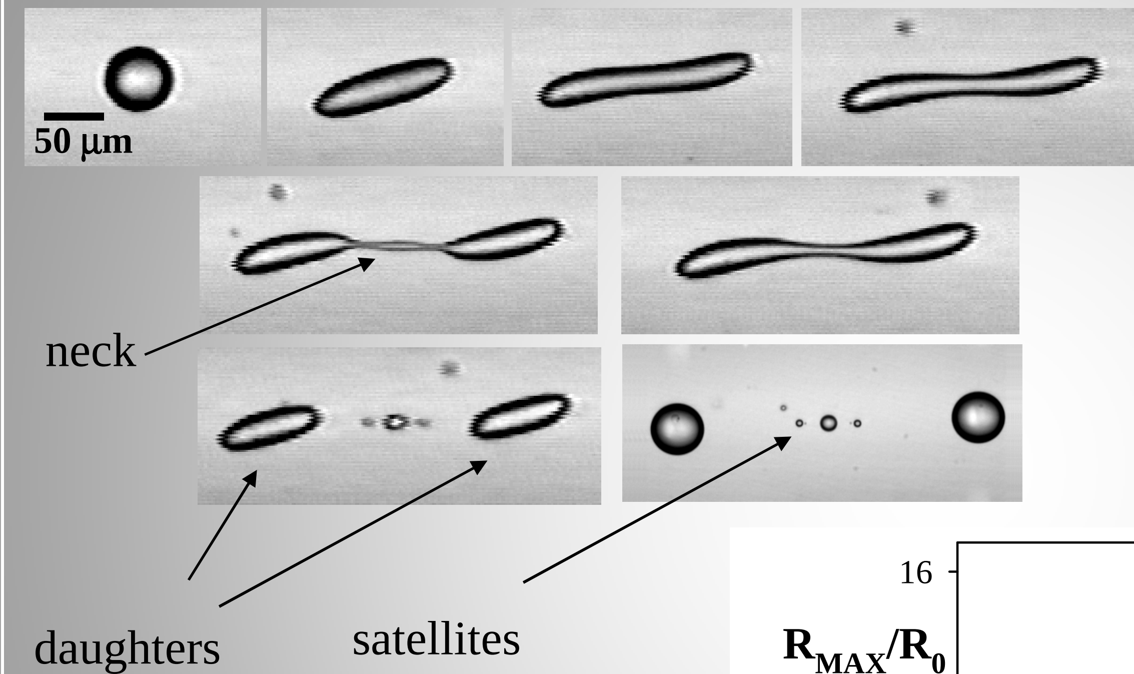
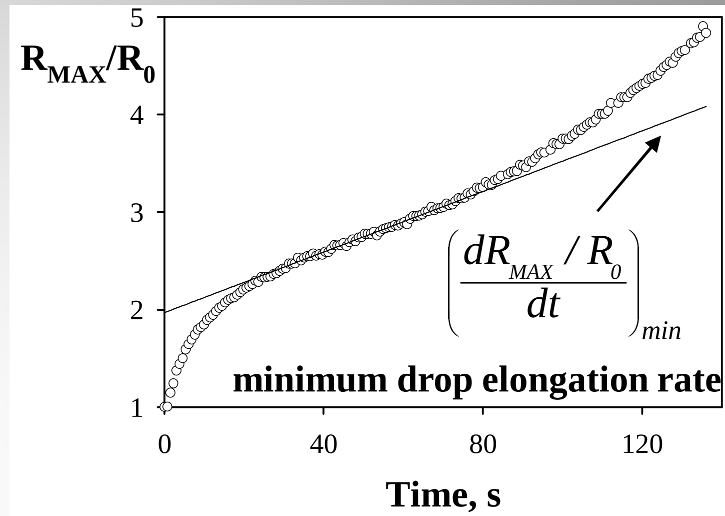
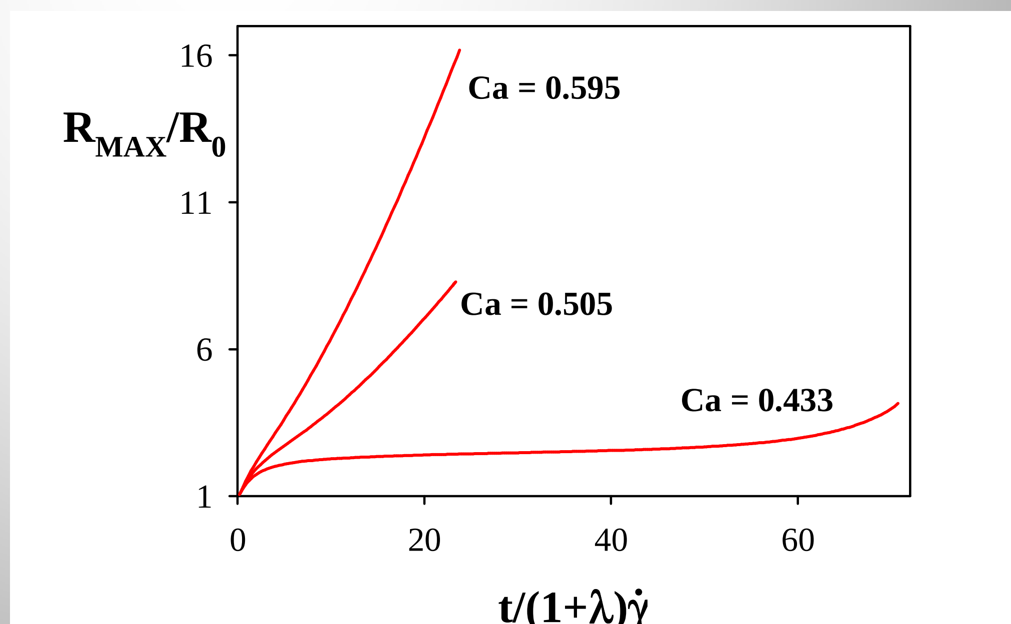
Breakup kinetics

$\lambda = 1 \qquad Ca = 0.50$







Cristini V, Guido S, Alfani , Blawzdziewicz J and Loewenberg M, *J. Rheol.*, **47,** 1283-1298 (2003)

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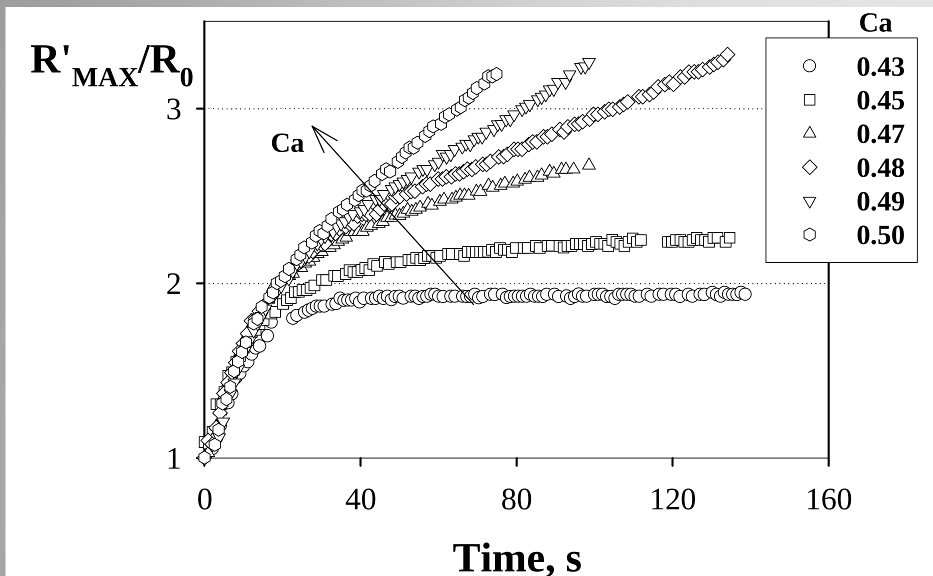
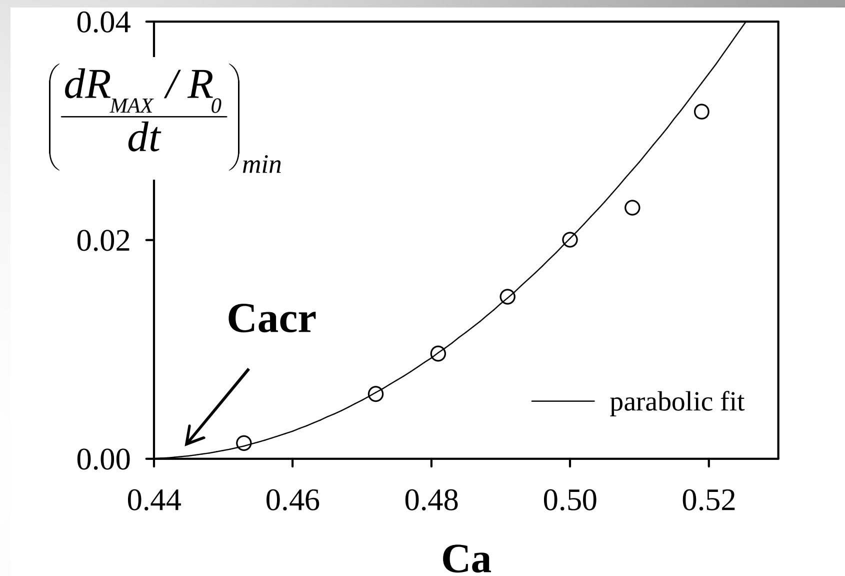
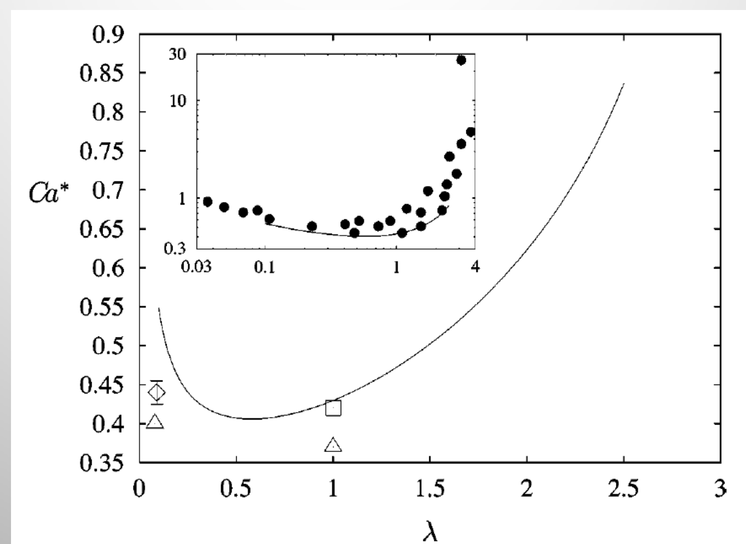
Determination of Ca_{cr}







(●) [Grace (1982)]

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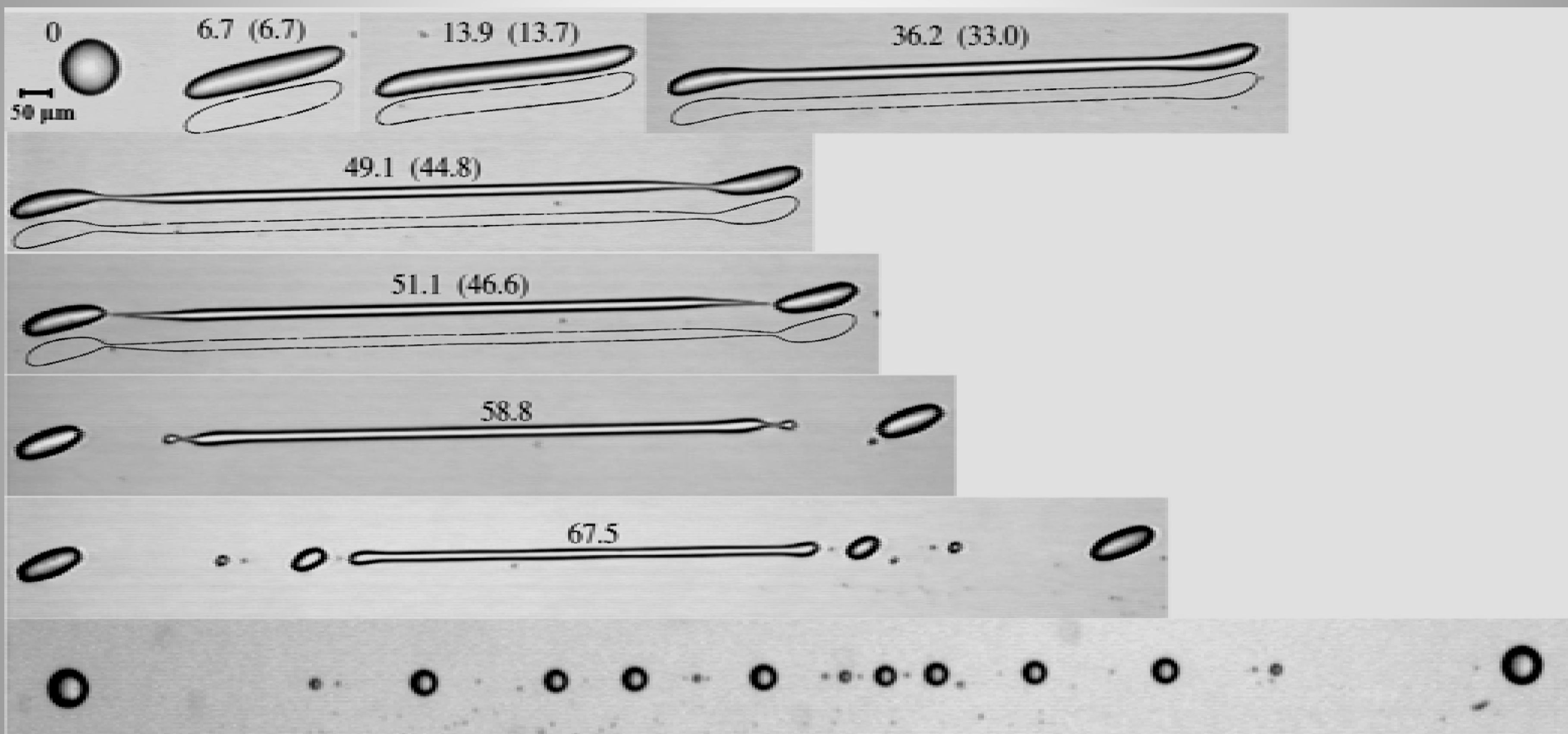
Shape evolution for Ca/Cacr = 1.38



Cristini V, Guido S, Alfani , Blawzdziewicz J and Loewenberg M, *J. Rheol.,* **47,** 1283-1298 (2003)

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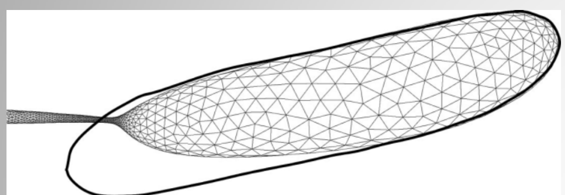
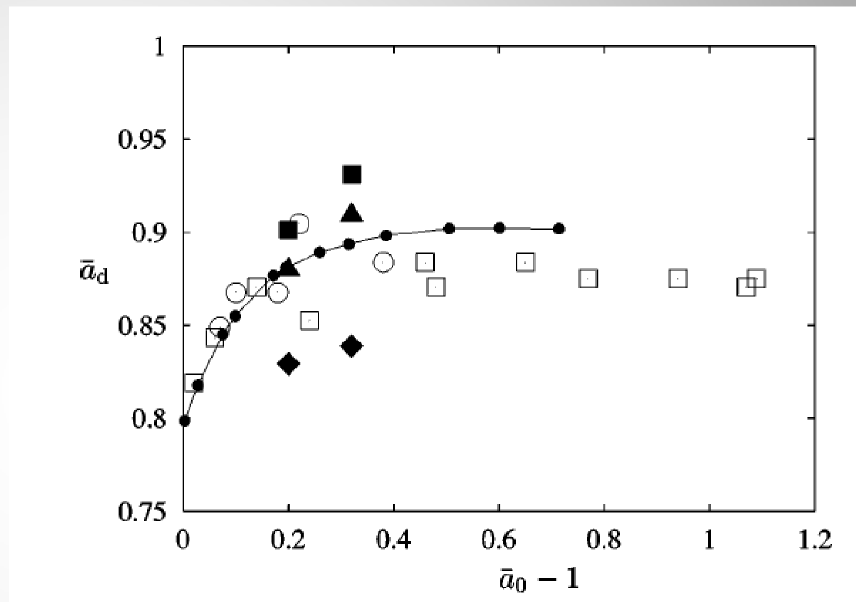
Daughter drop scaling





λ = 0.094 (□), λ = 0.5 (△), λ = 1.02 (○), λ = 2 (◇)

Daughter drop size — **indipendent of initial size** / **scales with critical drop size**

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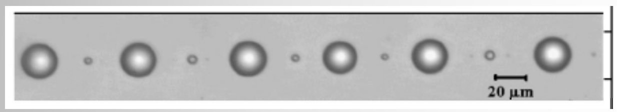
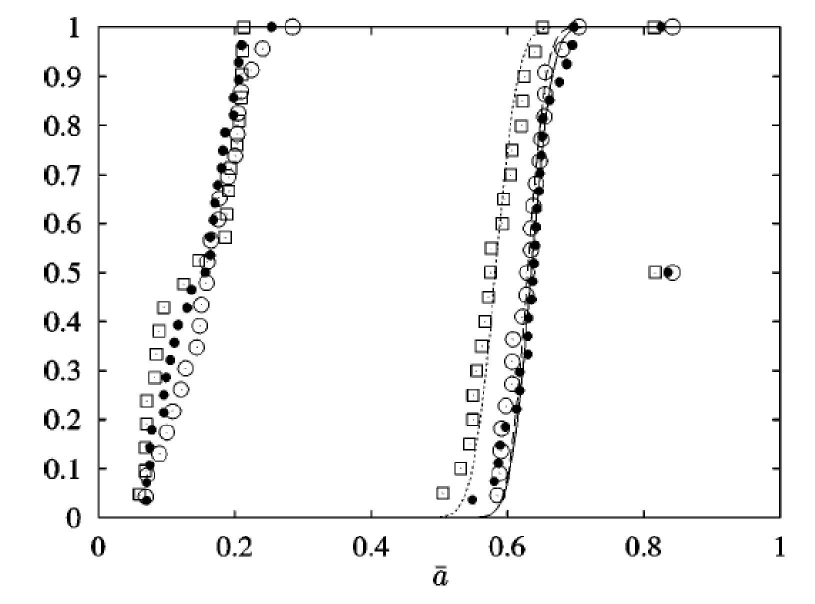
Drop fragment distribution

Cumulative size distribution



The distributions for each experiment show two distinct daughter drops and three size classes of satellite drops



$\lambda = 0.094$; $\bar{a}_0 = 1.22$ (◇), $\bar{a}_0 = 1.44$ (△), $\bar{a}_0 = 1.73$ (□), $\bar{a}_0 = 1.90$ (○), $\bar{a}_0 = 2.02$ (●), $\bar{a}_0 = 2.04$ (⊗).

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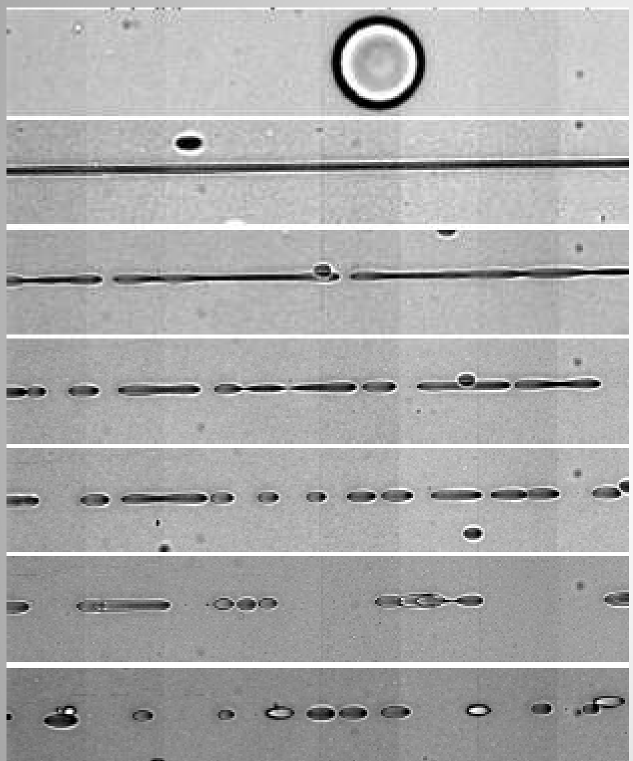
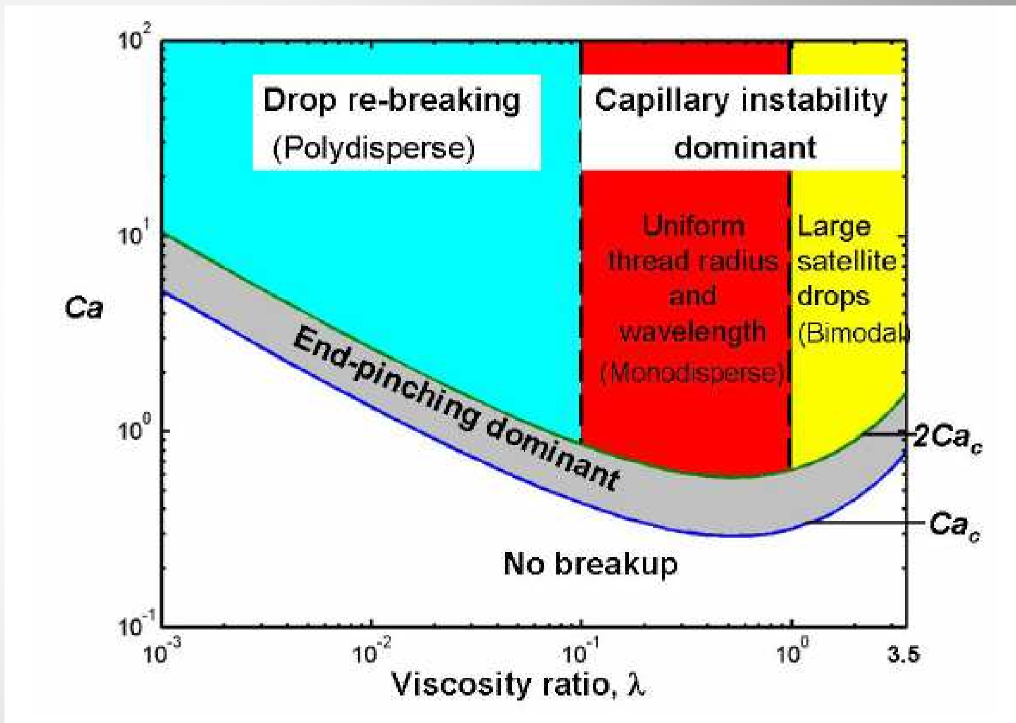
Drop fragment distribution



$\lambda = 0.075$, Ca = 4.5 Ca_{cr}



Zhao X, *J. Rheol.*, (2006)

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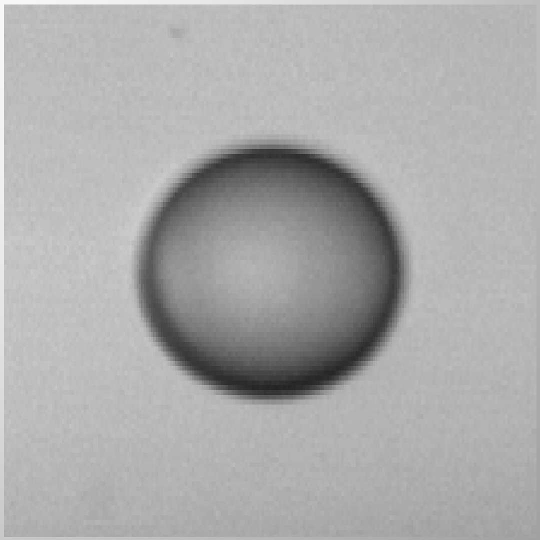
Non-Newtonian effects

- Liquid-liquid dispersions are *always* viscoelastic systems, due to interfacial tension
- Our aim is to study the effect of the *intrinsic elasticity* of the fluid components on flow-induced morphology
- Model fluids: constant viscosity, highly elastic liquids (Boger fluids)

Ca = 0.4

Outer phase: viscoelastic fluid , inner phase: Newtonian, λ = 1



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Small deformation theory with elastic fluids

- Constitutive equation of component liquids: second-order fluid

Here **either the outer or the inner phase is non-Newtonian (the other being Newtonian)** ⇒ 2 additional physical quantities:

normal stress differences

$$N_1 = \Psi_1 \dot{\gamma}^2$$

$$N_2 = \Psi_2 \dot{\gamma}^2$$

- New nondimensional parameters:

$$\mu = \frac{\Psi_2}{\Psi_1} \qquad W = \frac{r_0 \Psi_1 \dot{\gamma}^2}{2\sigma}$$

- Small deformation conditions $Ca \ll 1$ $W \ll 1$

Observable non-Newtonian effects ⇔

$$p \equiv \frac{W}{Ca^2} = \frac{\psi_1 \sigma}{2 r_0 \eta^2} \approx 1$$

Greco F, *J. Non-Newt. Fluid Mech.*, **107**, 111-131 (2002)

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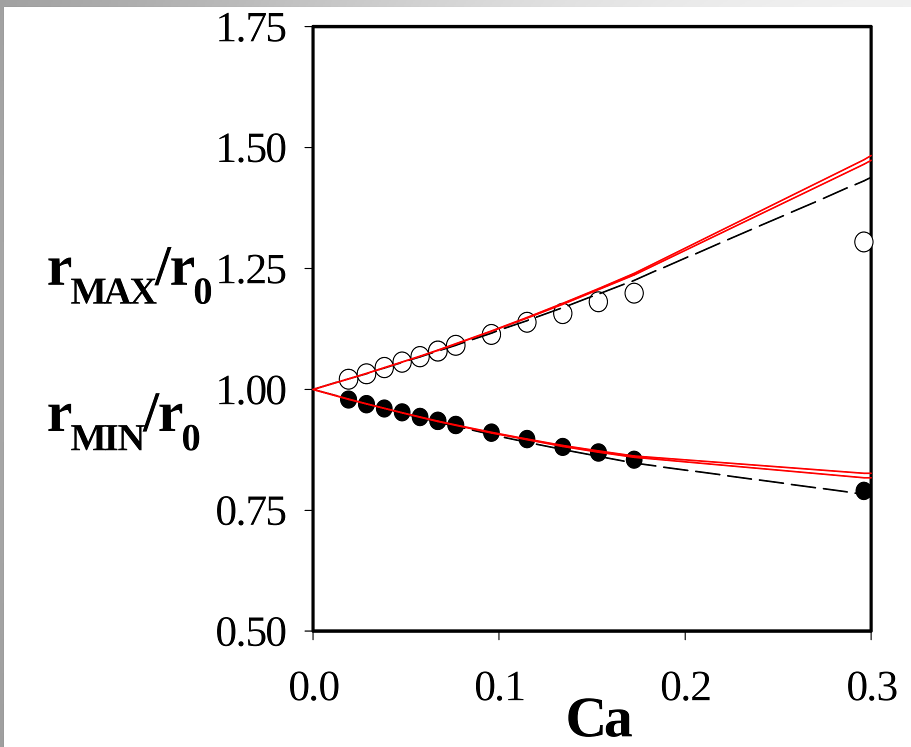
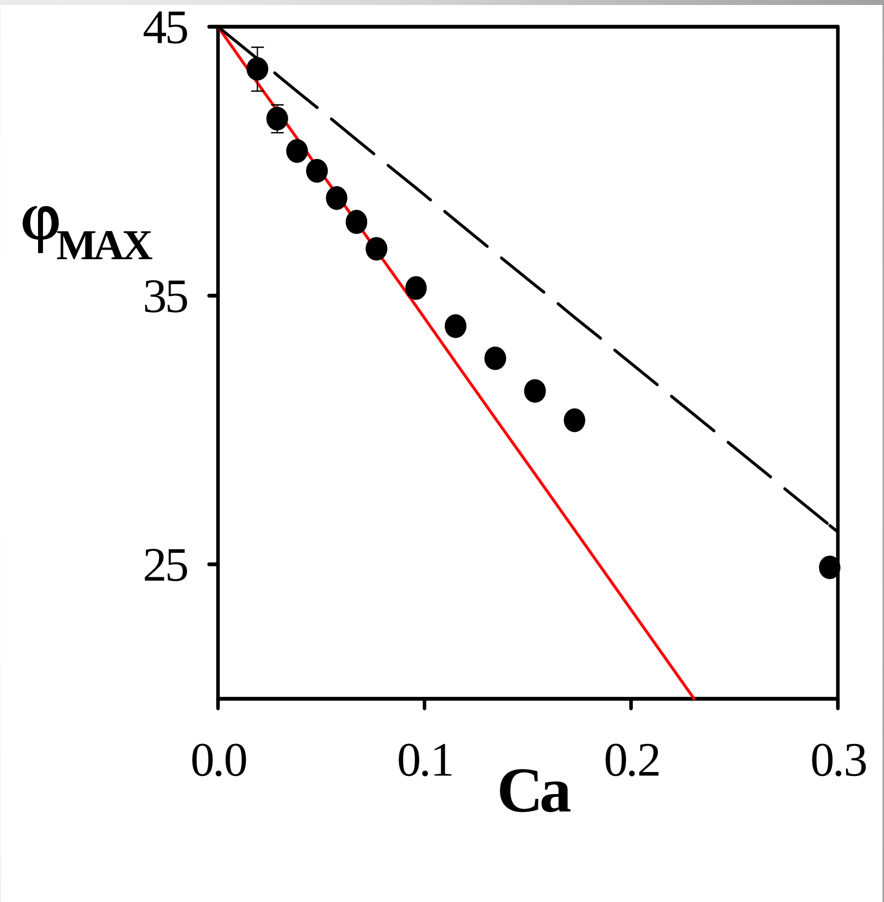
Comparison with experiments





$\lambda = 1$

p = 1.8

Dashed: Newtonian

Continuous: non-Newtonian

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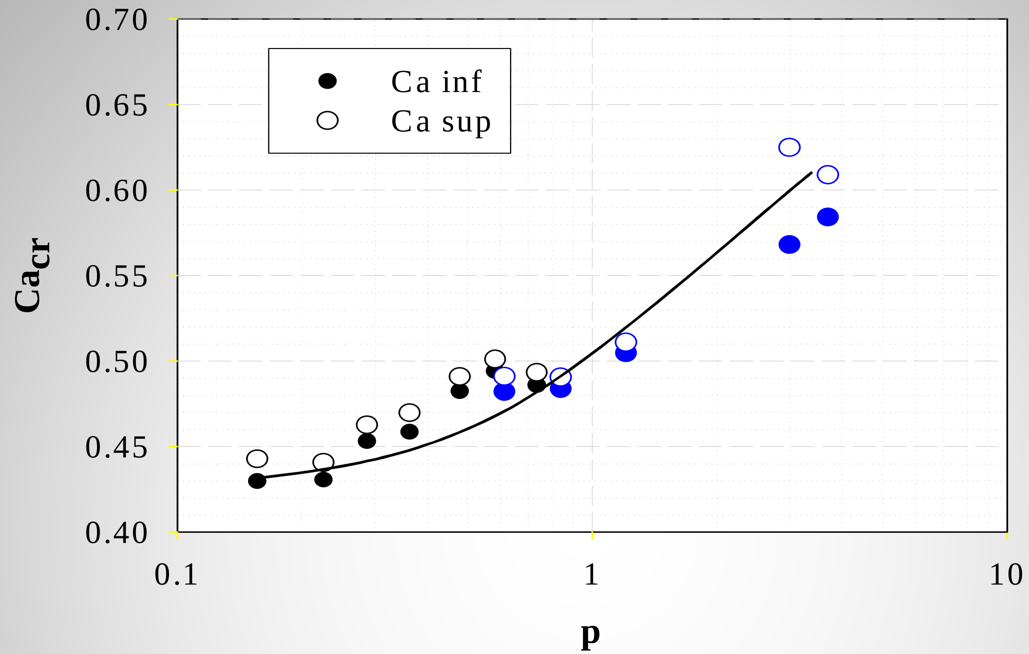
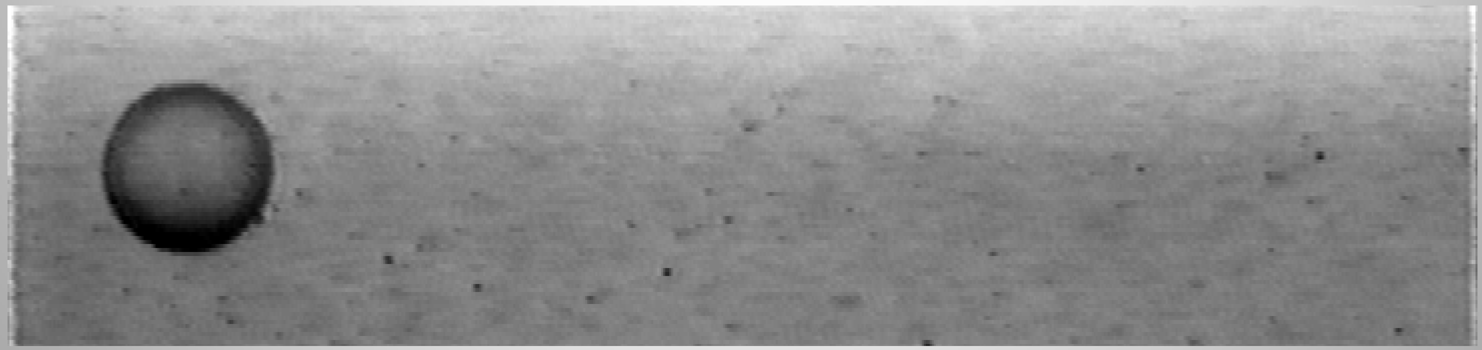
Breakup



Outer phase: viscoelastic, drop: Newtonian, λ = 0.6



Outer phase: Newtonian, drop: viscoelastic, λ = 2.6

Drop breakup is hindered by elasticity of the fluid components

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Wall effects

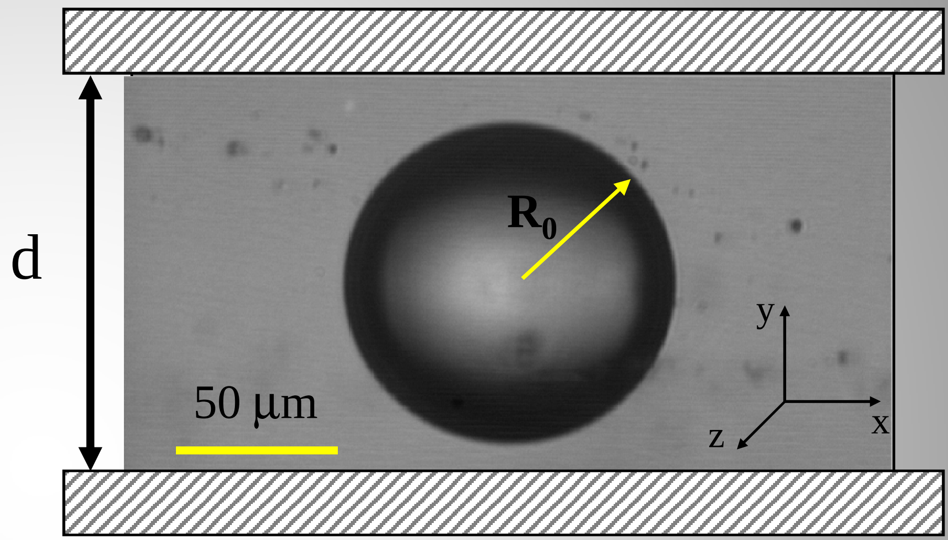
Shape parameters

$$\frac{L}{2R_0}$$

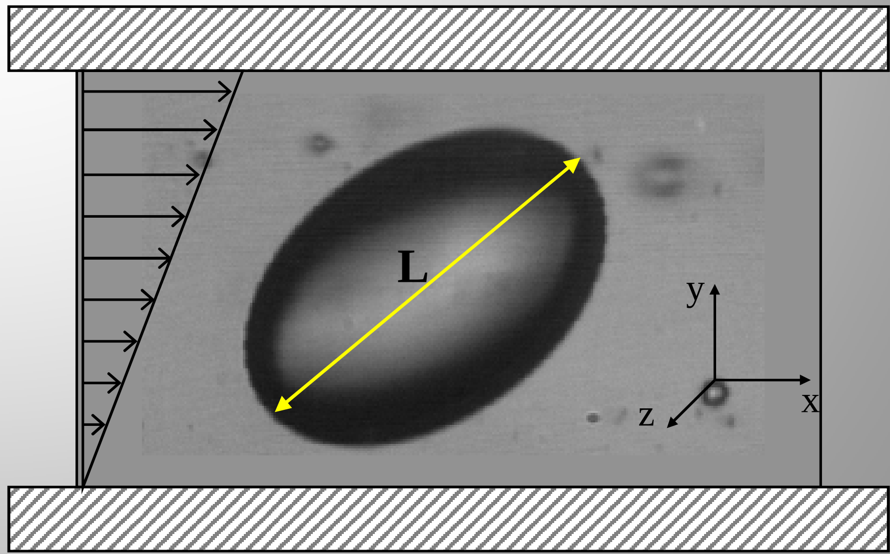
Nondimensional major axis

$$\frac{d}{2R_0}$$

Nondimensional gap



$$D = D_T\left[1+\left(\frac{2R_0}{d}\right)^3\frac{1+2.5\lambda}{1+\lambda}\right]$$



Shapira M and Haber S, *Int. J. Multiphase Flow*, **16,** 305 (1990)

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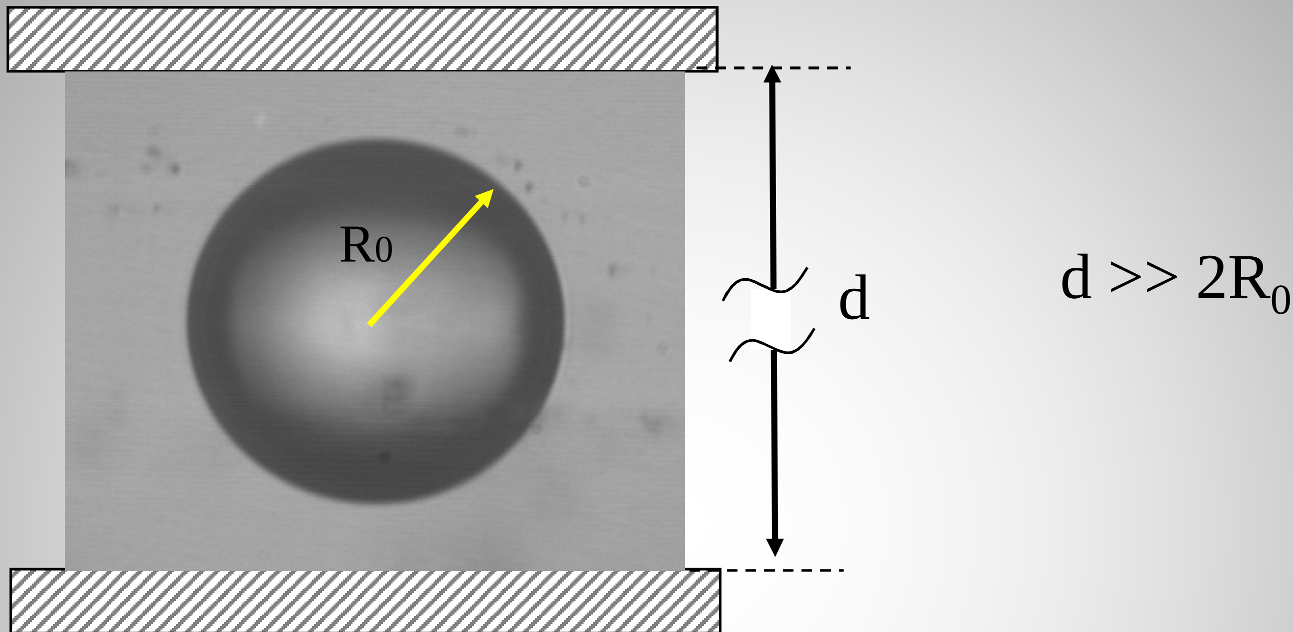
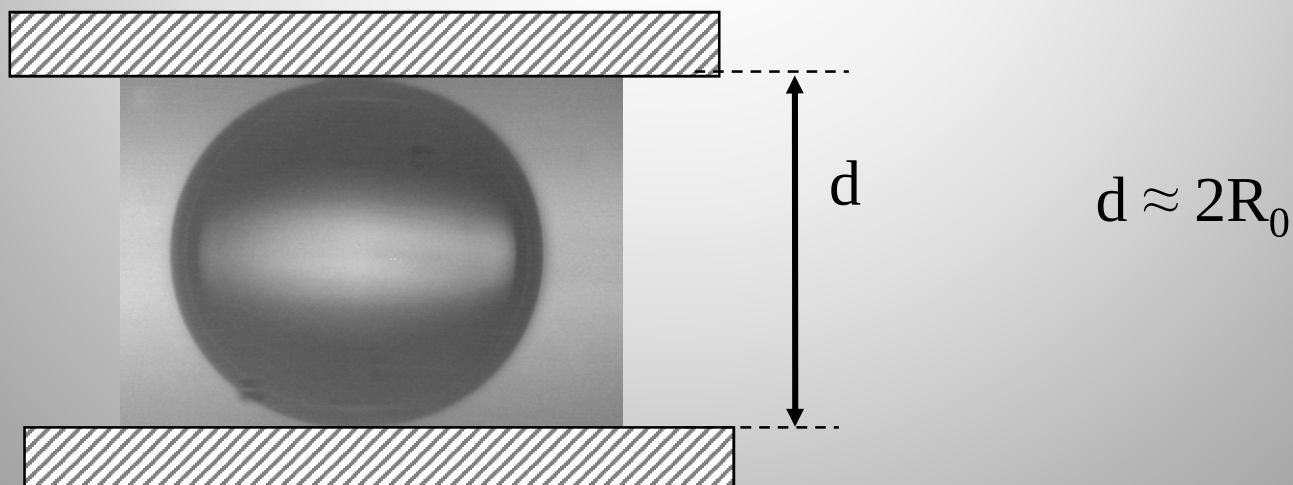
Wall effects





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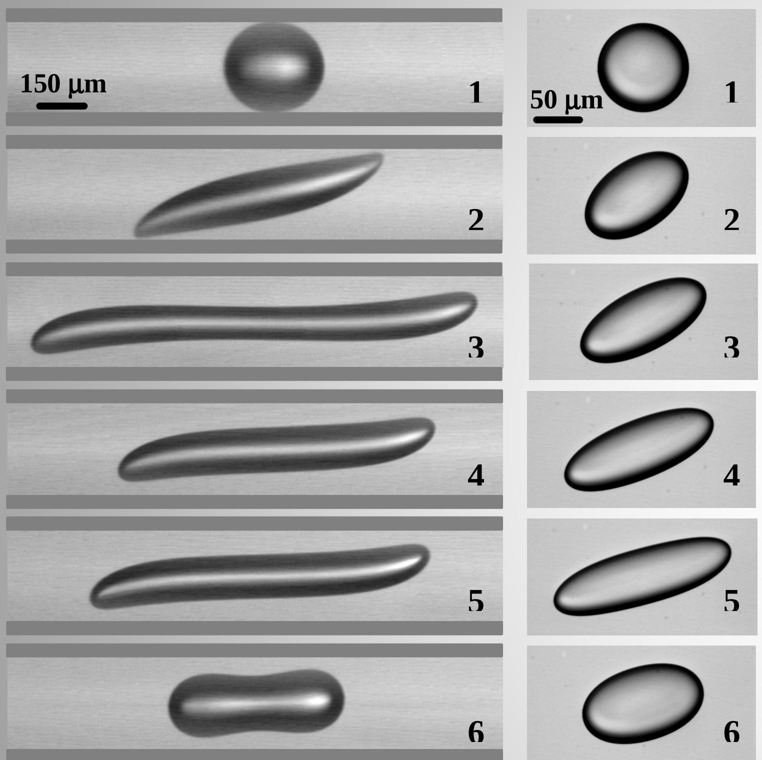
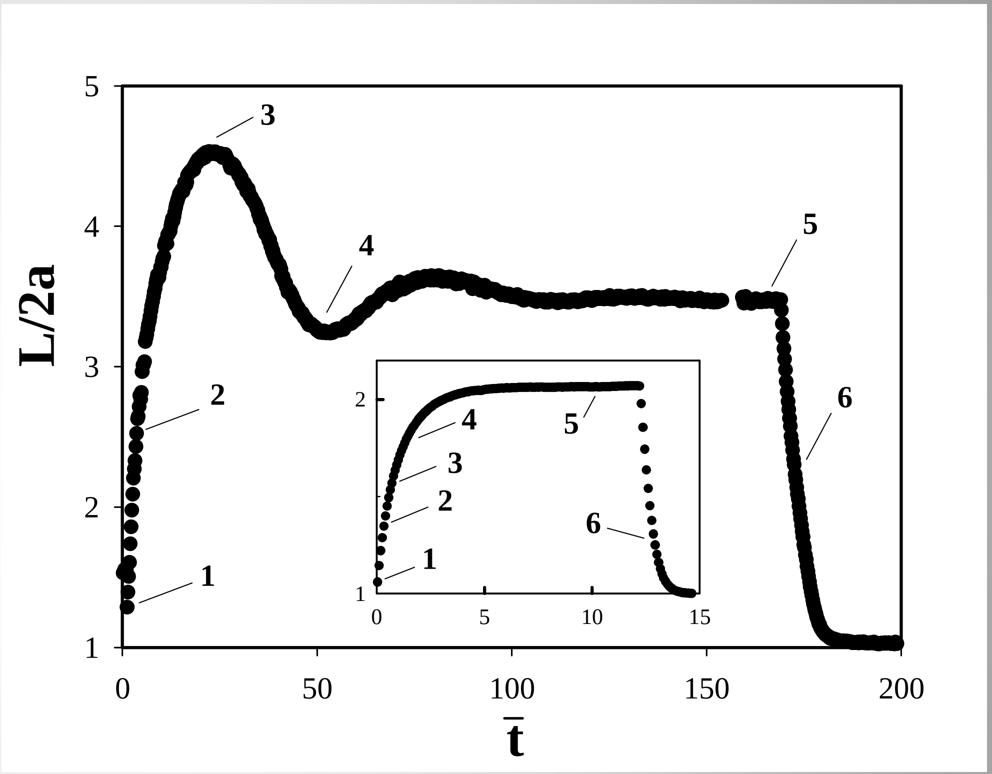


$d \approx 2R_0$ $d >> 2R_0$



Ca = 0.4, λ = 1

Sibillo V, Pasquariello G, Simeone M, Cristini V, Guido S, *Phys. Rev. Lett.*, **97**, 054502 (2006)

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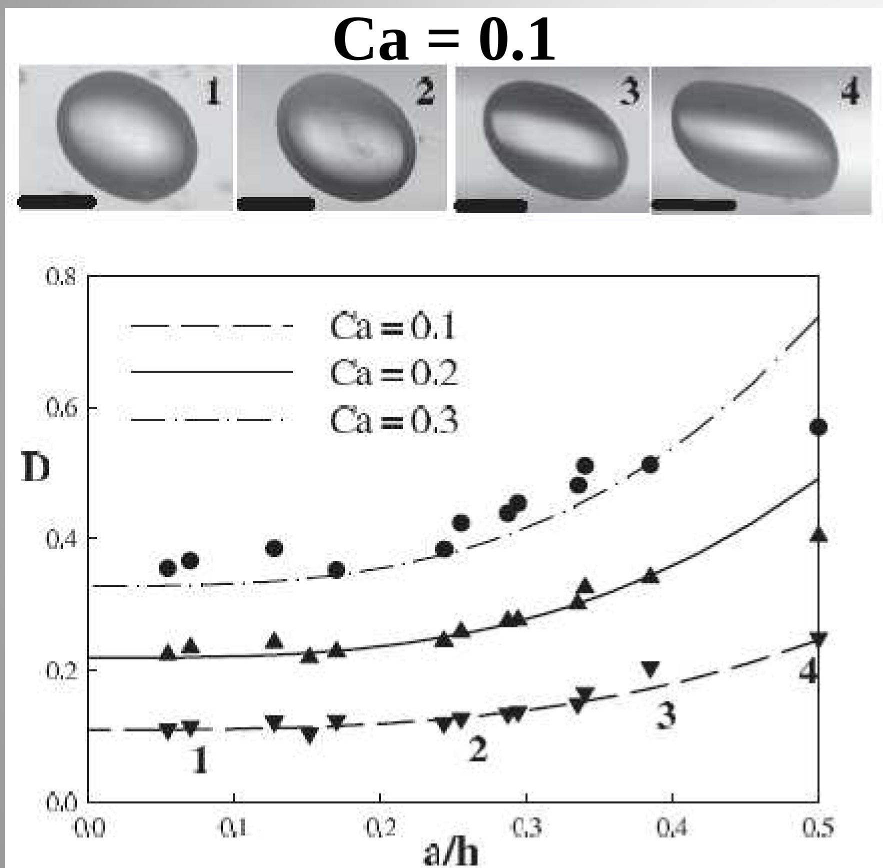
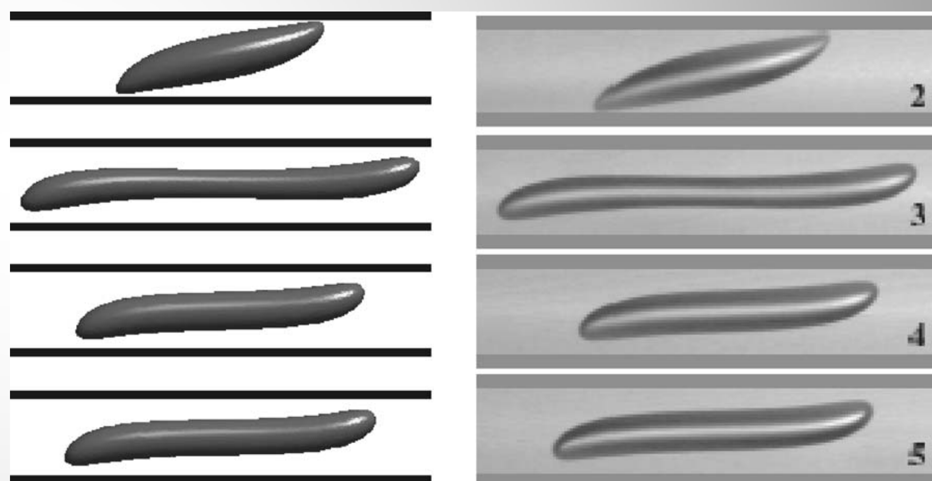
Wall effects

Comparison with theoretical predictions (left) and numerical simulations (right)



Theory: Shapira M and Haber S, *Int. J. Multiphase Flow*, **16,** 305 (1990)



Simulations: Janssen P J A and Anderson P D, Phys Fluids, 19, 043602 (2007)

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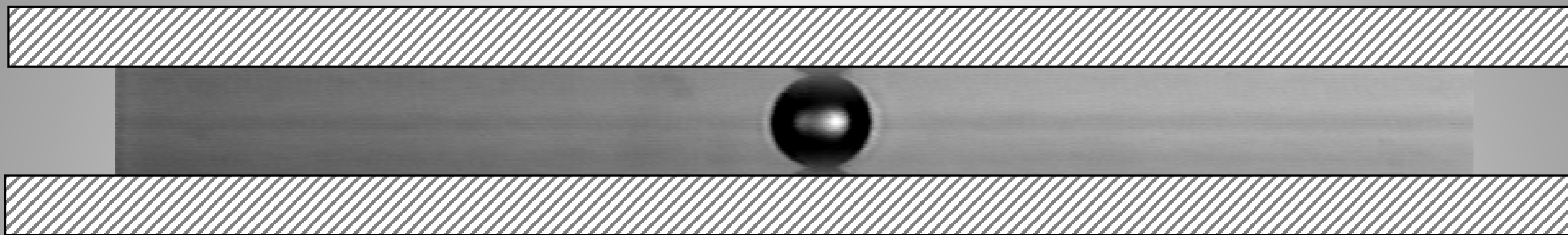
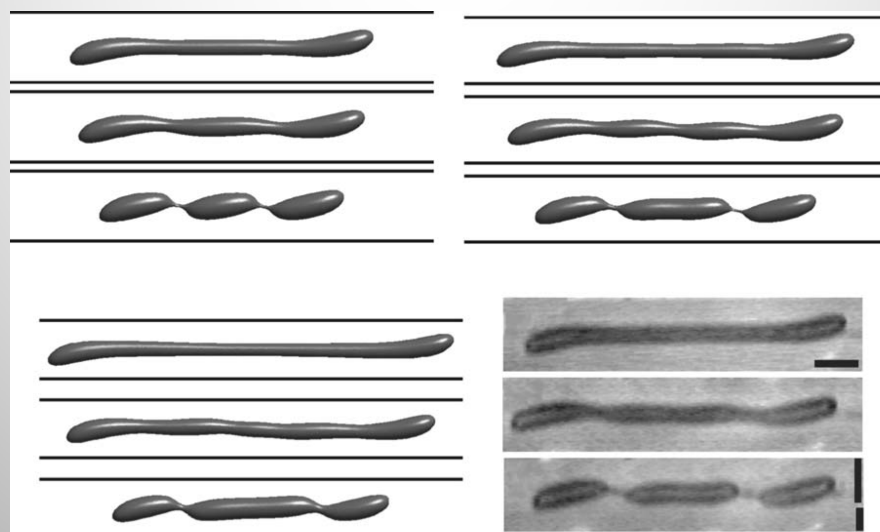


Wall effects act to stabilize drop shape at $\lambda = 1$



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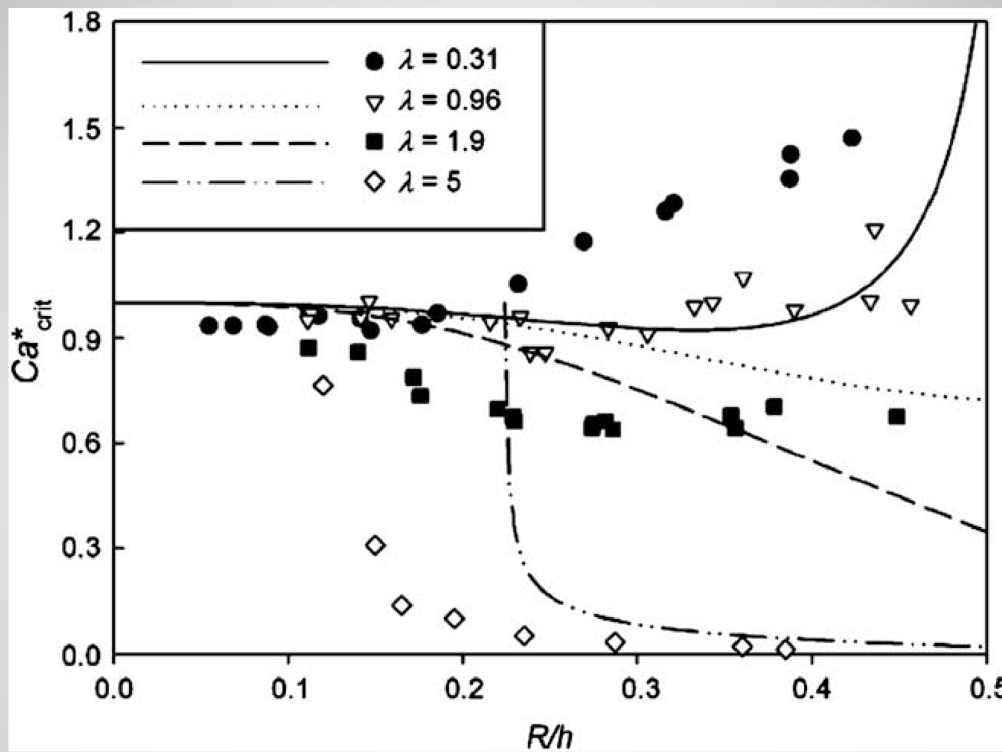
Wall effects



Even at λ > 4 (no breakup in unbounded shear flow), droplets can still be broken in confined conditions

Van Puyvelde P, Vananroye A, Cardinaels R, Moldenaers P, *Polymer*, **49**, 5363–5372 (2008)

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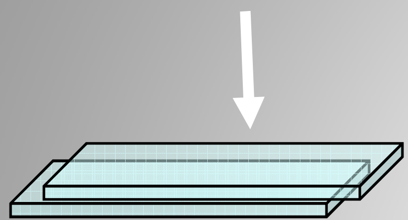
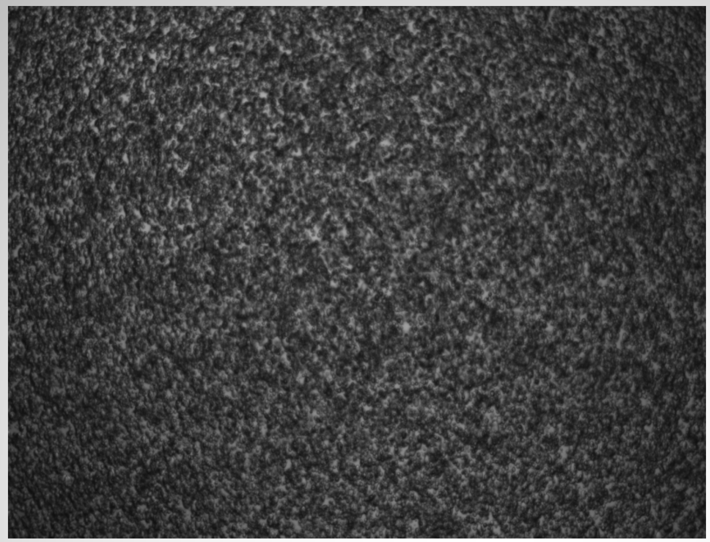
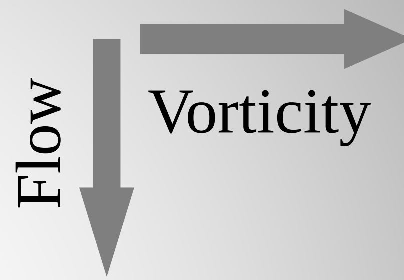
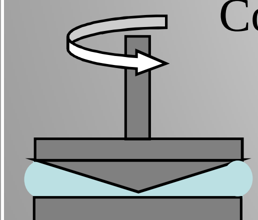
Wall effects: Shear banding

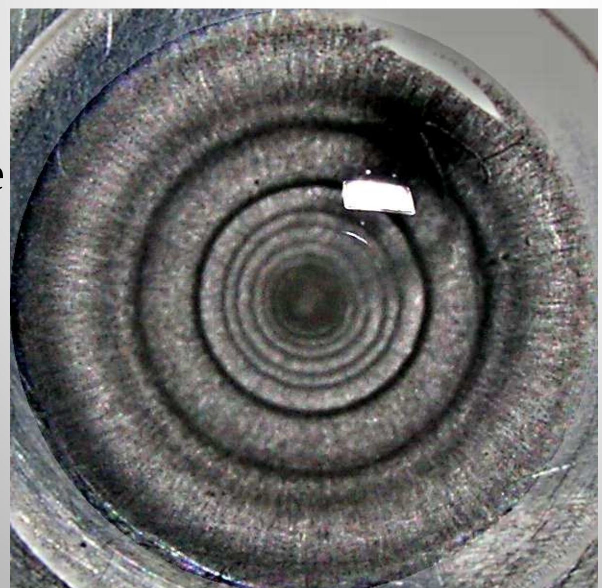


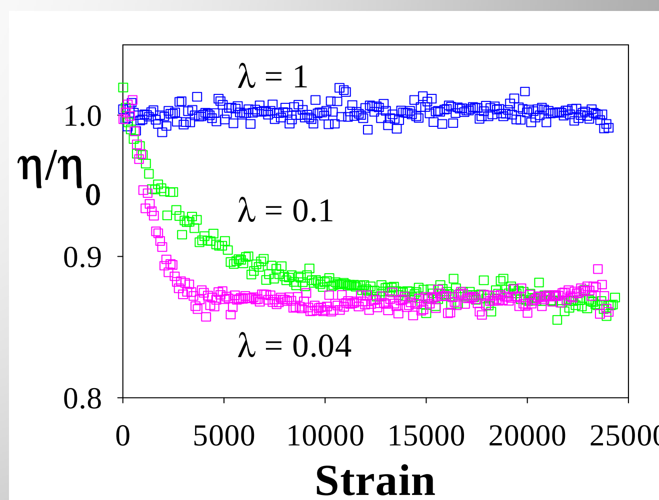






Cone and plate rheometer





Caserta S, Simeone M, and Guido S, *Phys. Rev. Lett.*, **100**, 137801 (2008)

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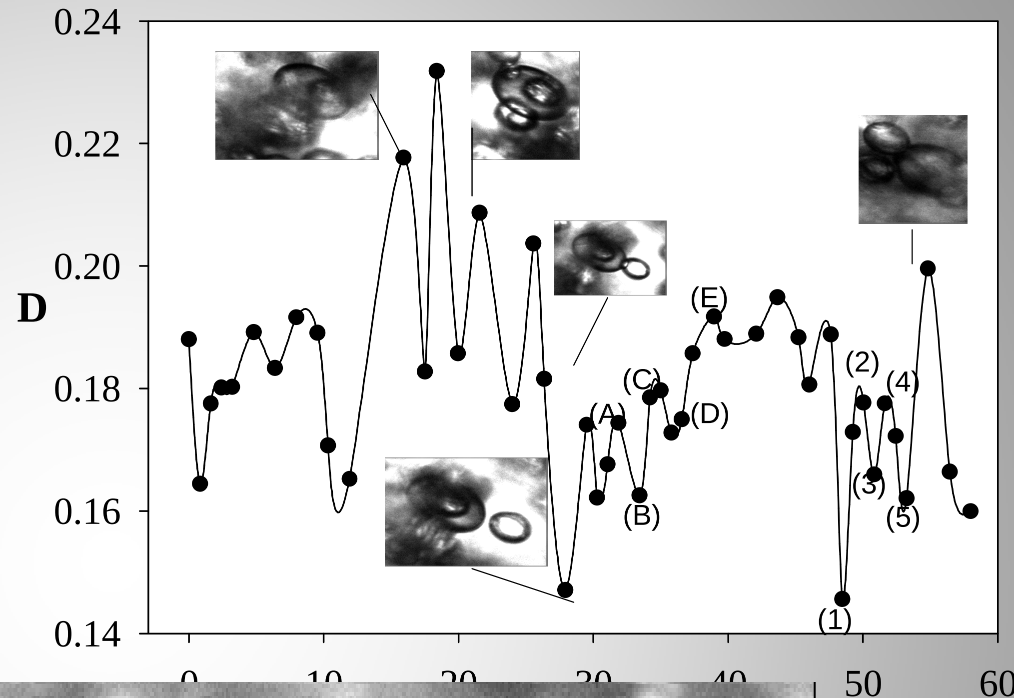
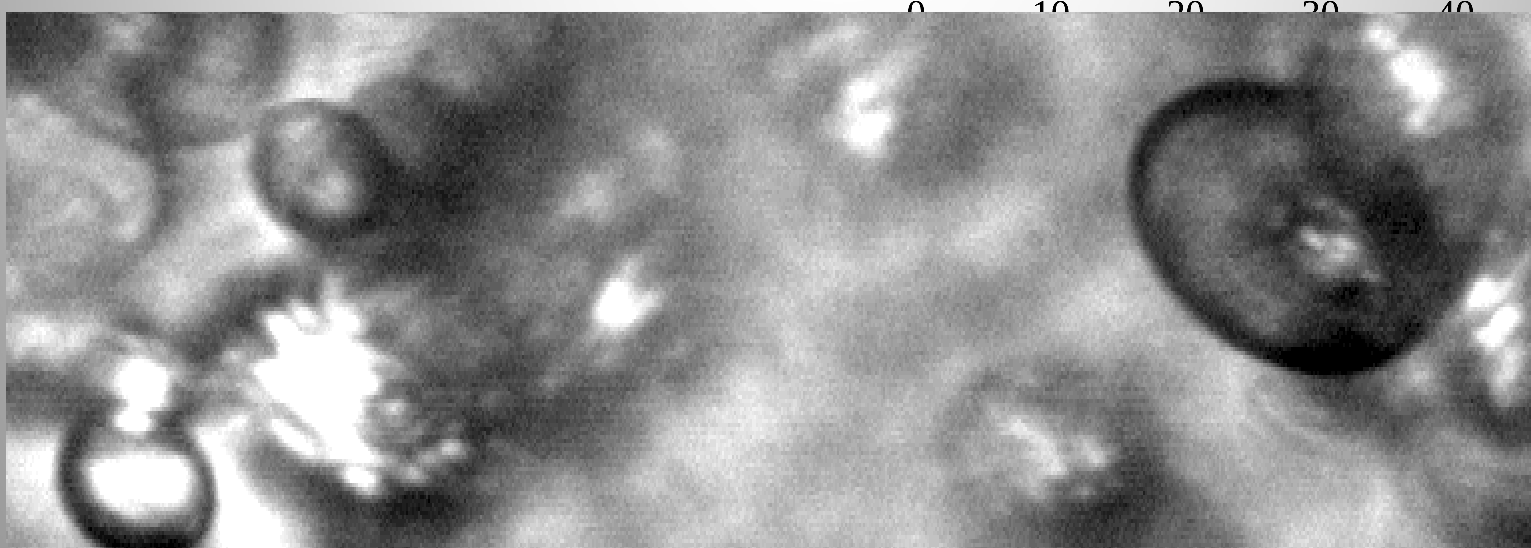
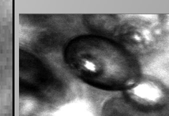
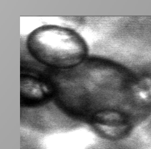
Concentrated systems

Shape fluctuations due to drop interactions






(E)


(5)

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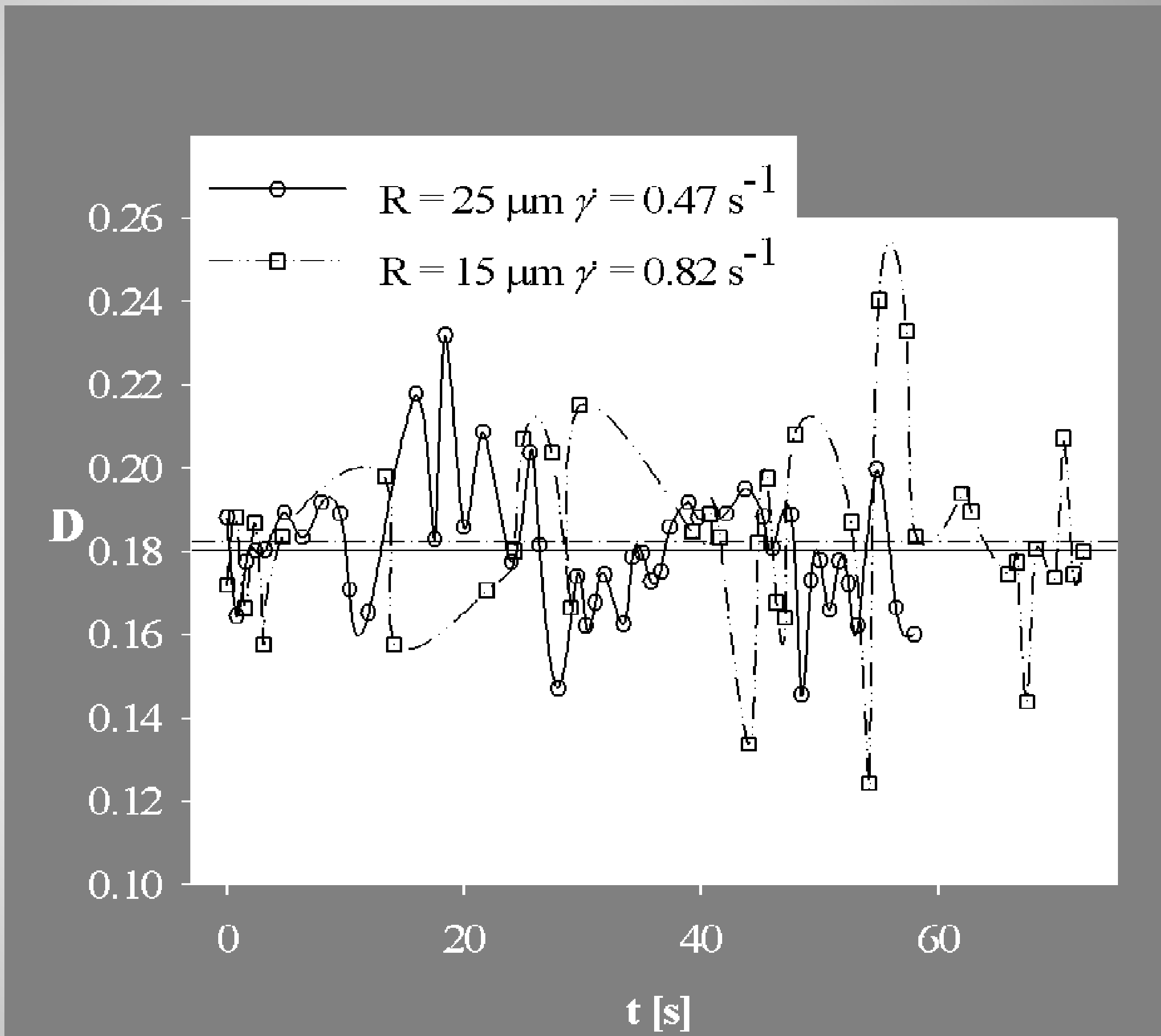
Concentrated systems

Ca = 0.15



The time-averaged value of D depends on Ca only

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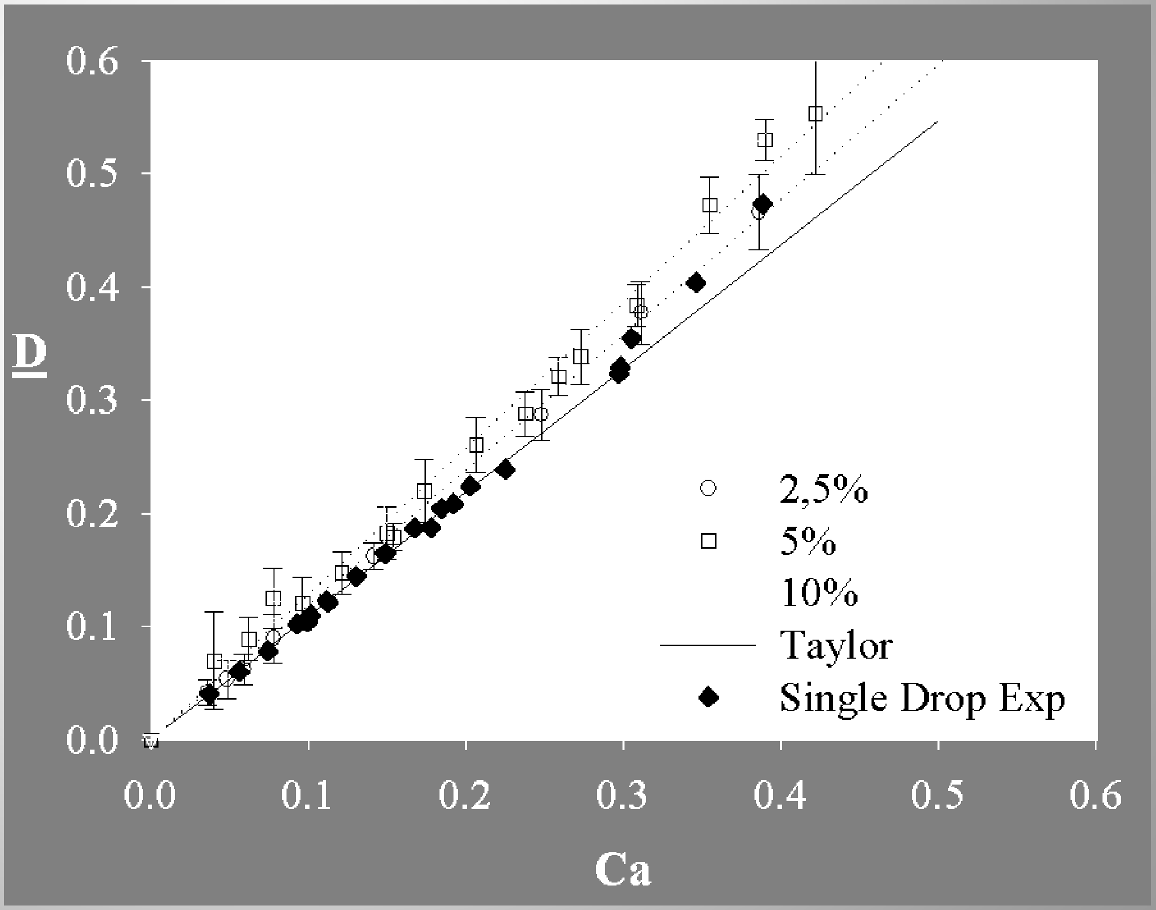
Concentrated systems



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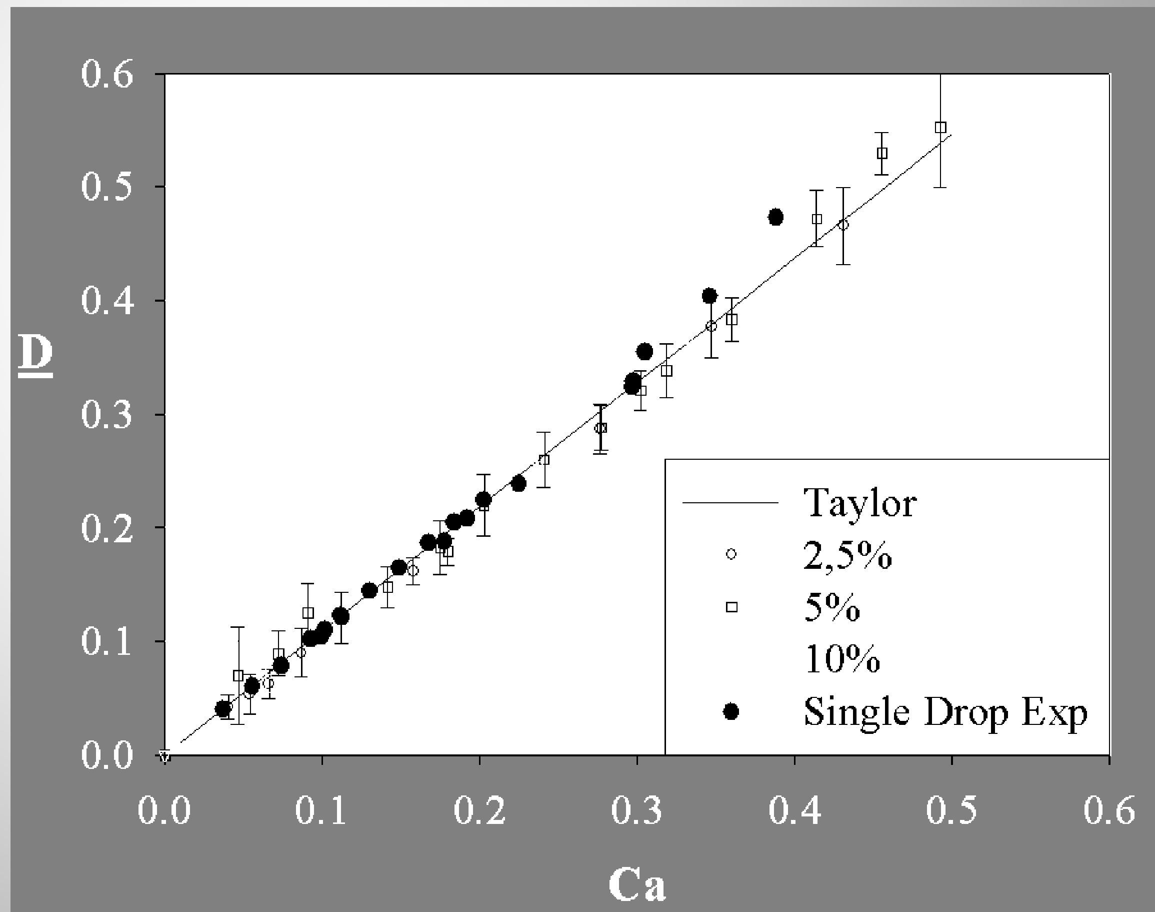
Concentrated systems

"Mean field" scaling with blend viscosity



Jansen K M B, Agterof W G M, Mellema J, *J Rheol*, **45**, 227-236 (2001)
Caserta S, Reynaud S, Simeone M and Guido S, *J Rheol*, **51**, 585-774 (2007) (data shown here)

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Conclusions

- Drop shape up to moderate deformations is essentially ellipsoidal and is well represented by small deformation theories and phenomenological models
- Numerical simulations are in good agreement with experiments up to breakup
- Drop fragments distribution can be estimated if the original distribution is known
- Single drop deformation and breakup results can be applied to concentrated systems by using a “mean field” scaling
- Wall effects stabilize drop shape and elicit shear banding phenomena
- Open issue: effects of surfactants